The CAPM
(Capital Asset Pricing Model)

NPV Dependent on Discount Rate Schedule

- Discussed NPV and time value of money
- Choice of discount rate influences decisions
- WACC may be appropriate for average projects
- What discount rate applies to unique projects?
CAPM: A Basis for Adjusting Discount Rates for Risk

- Development of the Capital Asset Pricing Model
  - Assumptions about investor’s view of risky investments
  - Risk characteristics and components
  - Principle of diversification
  - Beta: a formal metric of risk
  - The Capital Asset Pricing Model relationship between risk and expected return
  - The Security Market Line and expected return for individual investments

- Use of CAPM principles for project evaluation

- Comparison of utility theory and CAPM

Motivation for CAPM:
Investors Prefer Less Risk

- Consider two investments
  - Deposit $10 in a savings account with annual yield of 5%
  - Buy stock for $10 with a 50 - 50 chance of selling for $12 or $9 in one year

- Which is more attractive to risk-averse investors?
  - Expected return for savings account = 5%
  - Expected return for stock = \((0.5*(12+9)-10)/10\)*100%\(= 5\%

- For same return, investors prefer less risky savings account

- What if stock had a 75% chance of selling for $12?
Motivation for CAPM (2)

How do Investors Regard Risk and Return?

- Two key observations regarding preferences

  - Non-satisfaction
    - For a given level of risk, the preferred alternative is one with the highest expected return ($A > C$)

  - Risk Aversion
    - For a given level of return, the preferred alternative is one with the lowest level of risk ($A > B$)
Risk Metrics: An Empirical Observation

- Investors expect compensation for variability (risk)
- Risk-free rate defined as return if no variability
- More risky securities priced to return premium
- Correlation between variability and expected return

<table>
<thead>
<tr>
<th>Security</th>
<th>Expected Return</th>
<th>Variability: Standard Deviation of Expected Returns (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk free</td>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>U.S Treasuries</td>
<td>7.7</td>
<td>3.3</td>
</tr>
<tr>
<td>Fixed Income</td>
<td>9.0</td>
<td>9.0</td>
</tr>
<tr>
<td>Domestic Equity</td>
<td>12.7</td>
<td>18.5</td>
</tr>
<tr>
<td>International Equity</td>
<td>12.9</td>
<td>19.4</td>
</tr>
<tr>
<td>Real Estate</td>
<td>12.9</td>
<td>16.9</td>
</tr>
<tr>
<td>Venture Capital</td>
<td>18.6</td>
<td>33.0</td>
</tr>
</tbody>
</table>

Relationship Between Variability and Expected Return

- An upward trend
- Argument based on aggregate performance of groups
- CAPM models expectations for individual investments
Components of Risk

- Finance defines 2 risks (standard deviation)
- **Market Risk** (systematic, non-diversifiable)
  - Investments tend to fluctuate with outside markets
  - Declines in the stock market and the price of Microsoft might be correlated
- **Unique or Project Risk** (idiosyncratic, diversifiable)
  - Individual characteristics of investments affect return
  - Microsoft might increase in price, despite a decline in the overall stock market
- Diversify by holding a portfolio of many investments
- What compensation should investors demand for each type?

Role of Diversification

- Unique risks are reduced by holding an investment portfolio

- Example: Two Stocks
  - A: Expected return = 20%, Standard Deviation of Expected Returns = 20%
  - B: Expected Return = 20%
    - Standard Deviation of Expected Returns = 20%
- Consider portfolio with equal amounts of A and B
  - Expected return = 0.5*20% + 0.5*20% = 20%
  - Standard Deviation?
Standard Deviation for a Portfolio

- Portfolio standard deviation is not a weighted average

- Portfolio standard deviation

\[ \sigma_p = \sqrt{\sum_i \sum_j x_i x_j \sigma_i \sigma_j \rho_{ij}} \]

for a portfolio of N investments, with i, j = 1 to N

- \( x_i, x_j \) = Value fraction of portfolio represented by investments i and j
- \( \sigma_i, \sigma_j \) = Standard deviation of investments i and j
- \( \rho_{ij} \) = Correlation between investments i and j
- \( \rho_{jj} = 1.0 \)

Example: Standard Deviation for a 2 Stock Portfolio

- Invest equal amounts in two stocks
  - For both A & B: Expected Return = 20%, Standard Deviation = 20%

\[ \sigma_p = \sqrt{(0.5)(0.5)(0.2)(0.2)(1) + (0.5)(0.5)(0.2)(0.2)(1) + (2)(0.5)(0.5)(0.2)(0.2)\rho_{ab}} \]

- Portfolio standard deviation depends on correlation of A, B

<table>
<thead>
<tr>
<th>Correlation Between A &amp; B</th>
<th>Portfolio Standard Deviation</th>
<th>Portfolio Expected Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20.0%</td>
<td>20%</td>
</tr>
<tr>
<td>0.5</td>
<td>17.3%</td>
<td>20%</td>
</tr>
<tr>
<td>0</td>
<td>14.1%</td>
<td>20%</td>
</tr>
<tr>
<td>-1</td>
<td>0.0%</td>
<td>20%</td>
</tr>
</tbody>
</table>
Example: Standard Deviation for a 2 Stock Portfolio (2)

- Most investments not perfectly correlated (correlation < 1)
- Holding portfolio leads to risk reduction
- With negative correlation, can eliminate all risk

Generalization for Portfolio with Many Stocks

- General formula for standard deviation of portfolio returns
  \[ \sigma_p = \sqrt{\sum_{i} \sum_{j} x_i x_j \sigma_i \sigma_j \rho_{ij}} = \sqrt{\text{portfolio variance}} \]

- For a portfolio of N stocks in equal proportions \((x_i = x_j = 1/N)\)
  - N weighted variance terms, \(i = j \rightarrow \sigma_i^2\)
  - \((N^2-2-N)\) weighted cov. terms, \(i \neq j \rightarrow \sigma_i \sigma_j \rho_{ij}\)

- \(Var(P) = N^*(1/N)^2 \text{Average Variance} + (N^2-2-N)^*(1/N)^2 \text{Average Covariance}\)
- \(Var(P) = (1/N)^* \text{Av. Var.} + [1-(1/N)^*] \text{Av. Cov.}\)
Generalization for Portfolio with Many Stocks (2)

\[ \sigma_p = \sqrt{\frac{1}{N} \text{ Average Variance} + (1-\frac{1}{N}) \text{ Average Covariance}} \]

- For large \( N \), \( \frac{1}{N} \Rightarrow 0 \)
  - Average variance term associated with unique risks becomes irrelevant
  - Average covariance term associated with market risk remains

---

Defining a Formal Measure of Risk

- Investors expect compensation for systematic, undiversifiable (market) risk
- Standard deviation of returns reflects market & unique risks
- Need method to extract market portion of risk
- Define a reference point: the market portfolio
  - The full set of available securities
  - \( r_m = \text{Expected return for market portfolio} \)
  - \( \sigma_m = \text{Standard deviation of expected returns on market portfolio} \)
- Beta: index of investment risk compared to market portfolio
  - \( \beta_i = \rho_{i,m} \sigma_i / \sigma_m \)
What Does Beta Imply?

- By definition, the market portfolio has beta = 1.0
- Beta describes the relative variability of returns
  - Concerned with correlated (systematic) portion of returns
  - If investment amplifies movements in market portfolio beta > 1
  - If attenuates, movements in market portfolio beta < 1
- Beta reflects market risk of an investment
  - Investors expect higher returns for increased market risk
  - Thus, higher returns for investments with higher betas
- Can be calculated for individual investments or portfolios
- Portfolio beta = weighted average of individual betas

Investment Portfolios and the Efficient Frontier

- Example demonstrated role of diversification
- Combinations of many securities result in optimum
  - Maximum return for given risk level
  - Minimum risk for given level of return
  - Assumes no borrowing or lending
- Sub-optimal combinations lie below, to right of frontier
Combining Risk-Free and Risky Investments

- For any combination of risk-free and risky investing
  - Investor can mix investments in portfolio and risk-free to achieve desired return
  - Expected return is weighted average of risk-free (Rf) and portfolio return (Rp)
  - Standard deviation of Rf = 0
  - $\sigma_{mix} = x_p \sigma_p$

CAPM: Selecting a Portfolio to Maximize Returns for Risk

- Infinite number of portfolios, even on efficient frontier
- Tangent point yields optimum
- CAPM shows expected return for investment combinations
Determining Expected Return for Individual Investments

- CAPM models maximized expected return
- Beta indexes risk of individual investment to market portfolio
- Market portfolio is tangent point in CAPM
- Relation between beta and individual expected return results

![Graph]

How Does Expected Return Relate to Beta?

- **Security Market Line (SML)**
  - \( R_p = R_f + B_p \times (R_m - R_f) \)
  - \( R_m - R_f \) is the market risk premium
  - \( B_p \) is the beta of the portfolio or investment to be evaluated
- For the market portfolio, \( B_m = B_p = 1 \)
  - Total expected return is \( R_m \)
- For other investments, expected return scales with \( B_p \)
Differences Between Borrowing and Lending Rates

- Not typical to have same rate for borrowing and lending
- Risk-free rate generally unattainable for small investors
- Adjustments to model possible, minor, illustrated below
- Point of tangency shifts

Implementing the CAPM: From Theory to Project Evaluation

- Relation between market risk and expected return
  - Investments have market risk and unique risk components
  - Market risk commands premium over risk-free rate
  - Unique risk is managed (averaged out) by diversification
- Project discount rate should be based on project beta
  - Investors can diversify away unique project risks
  - Adjustment apparent if project is carbon-copy of firm (McDonald’s #10,001) ==> WACC applies
- “Proper” adjustment not trivial on most projects
  - Consider past experiences, returns in comparable industries
  - Detail unique aspects of specific project
  - Apply information to adjust discount rate
A General Rule for Managers

- Portfolio theory translates to a simple rule for managers:
- Use risk adjusted discount rate to calculate NPV for projects,
- Accept all positive NPV projects to maximize value
  —Shareholders capable of diversifying unique risks by holding multiple assets
  —Positive NPV implies market risk in projects is expected to be compensated
  —If projects are properly valued, shareholder wealth is maximized

Limitations and Conflicts in Practice

- Estimating project beta may not be trivial
- Budget constraints conflict with positive NPV rule
- Employees worry about unique project risks
  —Career can be adversely affected by bad outcomes
  —Not always in a position to diversify (limited to few projects)
  —Issue might be addressed through proper incentives
- Reliance on past performance to dictate future choices
- Individuals and companies are often “risk positive”
  —Entrepreneurs
  —Sometimes only choice is bet the company
How does Utility Theory Compare with CAPM?

- **Utility**
  - Applies a single discount rate for time value
  - Adjusts for risk preference of decision-maker
  - Utility is bottom-up and focused on individual preferences

- **CAPM**
  - Adjusts discount rate for overall aversion to market risk
  - No adjustment for risk preferences of decision-maker
  - Based on top-down, aggregate perspectives

- **Utility and CAPM**
  - Both value risky opportunities, accounting for risk aversion
  - Under the right circumstances, should give same results
  - “No double counting”

Summary

- **CAPM adjusts discount rates for risk**
  - Models maximum expected return for level of risk
  - Based on observations of securities markets

- **Unique risks can be diversified**

- **Investors expect compensation for market risk**

- **Standard deviation of returns reflects both market & unique**

- **Beta is index of market part of investment risk**

- **Security Market Line relates expected return to beta**
  - \[ R_p = R_f + B_p(R_m - R_f) \]

- **Moving from theory to practice can be problematic**