Primitive Decision Models

- Still widely used
- Illustrate problems with intuitive approach
- Provide base for appreciating advantages of decision analysis

Payoff Matrix as Basic Framework

BASIS: Payoff Matrix

<table>
<thead>
<tr>
<th>Alternative</th>
<th>State of “nature”</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$S_1$ $S_2$ $\ldots$ $S_M$</td>
</tr>
<tr>
<td>$A_1$</td>
<td></td>
</tr>
<tr>
<td>$A_2$</td>
<td>Value of outcomes</td>
</tr>
<tr>
<td>$A_N$</td>
<td>$O_{NM}$</td>
</tr>
</tbody>
</table>
Primitive Model: Laplace (1)

- Decision Rule:
  a) Assume each state of nature equally probable => \( p_m = \frac{1}{m} \)
  b) Use these probabilities to calculate an “expected” value for each alternative
  c) Maximize “expected” value

Primitive Model: Laplace (2)

- Example

<table>
<thead>
<tr>
<th></th>
<th>( S_1 )</th>
<th>( S_2 )</th>
<th>“expected” value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_1 )</td>
<td>100</td>
<td>40</td>
<td>70</td>
</tr>
<tr>
<td>( A_2 )</td>
<td>70</td>
<td>80</td>
<td>75</td>
</tr>
</tbody>
</table>
Primitive Model: Laplace (3)

- Problem: Sensitivity to framing
  ==> “irrelevant alternatives

<table>
<thead>
<tr>
<th></th>
<th>S_{1A}</th>
<th>S_{1A}</th>
<th>S_2</th>
<th>&quot;expected&quot; value</th>
</tr>
</thead>
<tbody>
<tr>
<td>A_1</td>
<td>100</td>
<td>100</td>
<td>40</td>
<td>80</td>
</tr>
<tr>
<td>A_2</td>
<td>70</td>
<td>70</td>
<td>80</td>
<td>73.3</td>
</tr>
</tbody>
</table>

Maximin or Maximax Rules (1)

- Decision Rule:
  a) Identify minimum or maximum outcomes for each alternative
  b) Choose alternative that maximizes the global minimum or maximum
Maximin or Maximax Rules (2)

- Example:

<table>
<thead>
<tr>
<th></th>
<th>S₁</th>
<th>S₂</th>
<th>S₃</th>
<th>maximin</th>
<th>maximax</th>
</tr>
</thead>
<tbody>
<tr>
<td>A₁</td>
<td>100</td>
<td>40</td>
<td>30</td>
<td></td>
<td>2</td>
</tr>
<tr>
<td>A₂</td>
<td>70</td>
<td>80</td>
<td>20</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>A₃</td>
<td>0</td>
<td>0</td>
<td>110</td>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>

- Problems
  - discards most information
  - focuses in extremes

Regret (1)

- Decision Rule
  a) Regret = (max outcome for state i) - (value for that alternative)
  b) Rewrite payoff matrix in terms of regret
  c) Minimize maximum regret (minimax)
Regret (2)

Example:

<table>
<thead>
<tr>
<th></th>
<th>S_1</th>
<th>S_2</th>
<th>S_3</th>
</tr>
</thead>
<tbody>
<tr>
<td>A_1</td>
<td>100</td>
<td>40</td>
<td>30</td>
</tr>
<tr>
<td>A_2</td>
<td>70</td>
<td>80</td>
<td>20</td>
</tr>
<tr>
<td>A_3</td>
<td>0</td>
<td>0</td>
<td>110</td>
</tr>
</tbody>
</table>

Regret (3)

Problem: Sensitivity to Irrelevant Alternatives

<table>
<thead>
<tr>
<th></th>
<th>S_1</th>
<th>S_2</th>
<th>S_3</th>
</tr>
</thead>
<tbody>
<tr>
<td>A_1</td>
<td>100</td>
<td>40</td>
<td>30</td>
</tr>
<tr>
<td>A_2</td>
<td>70</td>
<td>80</td>
<td>20</td>
</tr>
</tbody>
</table>

NOTE: Reversal of evaluation if alternative dropped

Problem: Potential Intransitivities
**Weighted Index Approach (1)**

- **Decision Rule**
  
  a) Portray each choice with its deterministic attribute -- different from payoff matrix

  
  For example:

<table>
<thead>
<tr>
<th>Material</th>
<th>Cost</th>
<th>Density</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$50</td>
<td>11</td>
</tr>
<tr>
<td>B</td>
<td>$50</td>
<td>9</td>
</tr>
</tbody>
</table>

**Weighted Index Approach (2)**

- b) Normalize table entries on some standard, to reduce the effect of differences in units. This could be a material (A or B); an average or extreme value, etc.

  For example:

<table>
<thead>
<tr>
<th>Material</th>
<th>Cost</th>
<th>Density</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1.00</td>
<td>1.000</td>
</tr>
<tr>
<td>B</td>
<td>1.20</td>
<td>0.818</td>
</tr>
</tbody>
</table>

  c) Decide according to weighted average of normalized attributes.
Weighted Index Approach (3)

- Problem 1: Sensitivity to Normalization

Example:

<table>
<thead>
<tr>
<th>Normalize on A</th>
<th>Normalize on B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Matl $</td>
<td>Dens $</td>
</tr>
<tr>
<td>A 1.00</td>
<td>1.000</td>
</tr>
<tr>
<td>B 1.20</td>
<td>0.818</td>
</tr>
</tbody>
</table>

Weighting both equally, we have
A > B (2.00 vs. 2.018)  B > A (2.00 vs. 2.05)

Weighted Index Approach (4)

- Problem 2: Sensitivity to Irrelevant Alternatives

As above, evident when introducing a new alternative, and thus, new normalization standards.

- Problem 3: Sensitivity to Framing “irrelevant attributes” similar to Laplace criterion (or any other using weights)
Example from Practice

- Sydney Environmental Impact Statement
- 10 potential sites for Second Airport
- About 80 characteristics

- The choice from first solution
- ... not chosen when poor choices dropped
- ... best choices depended on aggregation of attributes
- Procedure a mess -- totally dropped

Summary

- Primitive Models are full of problems

- Yet they are popular because
  people have complex spreadsheet data
  they seem to provide simple answers

- Now you should know why to avoid them!