

Dynamic Strategic Planning

Utility Assessment

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Utility Assessment

- **Basic Axioms**
- **Example**
- **Interview Process**
- **Procedures**
 - **Conventional**
 - **New**
- **Discussion**

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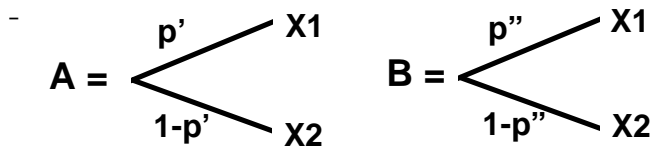
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Utility Function - U(X)

- **Definition:**
 - U(X) is a Special V(X),
 - Defined in an Uncertain Environment
- **It has a Special Advantage**
 - Units of U(X) DO measure relative preference
 - CAN be used in meaningful calculations

Basic Axioms of U(x)

- **Probability**
 - Probabilities exist - can be quantified
 - More is better



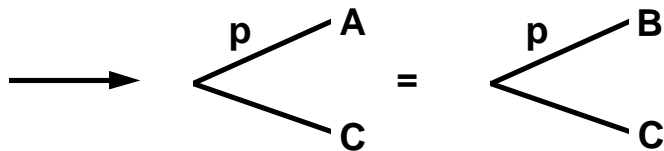
**If $X1 > X2$; $A > B$ if $p' > p''$
is preferred to**

Basic Axioms of $U(x)$ (cont'd)

- Preferences

- Linear in Probability

(substitution/independence) - Equals can be substituted if a subject is indifferent between A and B



Not a good assumption for small p (high consequences) !

Cardinal Scales

- Units of interval are equal, therefore averages and arithmetic operations are meaningful
- Two types exist
 - Ratio
Zero value implies an absence of phenomenon
e.g., Distance, Time
note: $F'(x) = a F(x)$
defines an equivalent measure (e.g., meters and feet)

Cardinal Scales (cont'd)

- **Ordered Metric**
Zero is relative, arbitrary
e.g., Temperature
define two points:
 - 0 degrees C - freezing point of pure water
 - 100 degrees C - boiling point of pure water at standard temperature and pressure
 - 0 degrees F - freezing point of salt water
 - 100 degrees F - What?**Note: $f'(x) = a f(x) + b$ (e.g. $F = (9/5) C + 32$)**
equivalent measures under a positive linear transformation

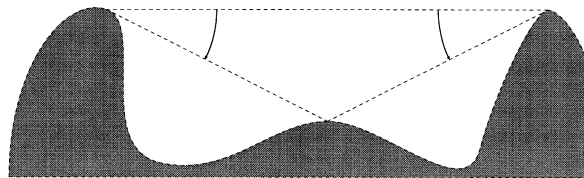
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equivalent measures under a positive linear transformation

Consequences of Utility Axioms

- **Utility exists on an ordered metric scale**
- **To measure, sufficient to**
 - Scale 2 points arbitrarily
 - obtain relative position of others by probability weightinge.g., Equivalent = $(X^*, p; X_*)$

Similar to triangulation in surveying



Interview Issues

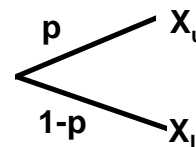
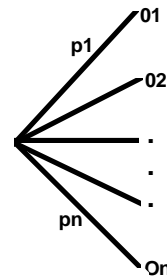
- **Put person at ease**
 - this individual is expert on his values
 - his opinions are valued
 - there are no wrong answers
 - **THIS IS NOT A TEST!!**
- **Scenario relevant to**
 - person
 - issues to be evaluated

Interview Issues (cont'd)

- Technique for obtaining equivalents:
BRACKETING
- Basic element for measurement:
LOTTERIES

Nomenclature

- **Lottery**
A risky situation with outcomes O_j
at probability p_j
Written as $(O_1, p_1; O_2, p_2; \dots)$
- **Binary Lottery**
A lottery with only two branches,
entirely defined by X_u, p_u, X_L
 $p(X_L) = 1 - P_u$
Written as $(X_u, P_u; X_L)$

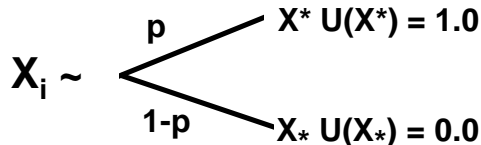


Nomenclature (cont'd)

- **Elementary Lottery**
Lottery where one outcome equals zero, i.e. status quo written as (X,p)

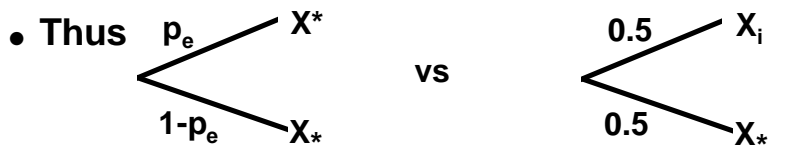
Utility Measurement

- **Conventional Method**
- **Certainty Equivalent - Balance X_i and a lottery**
 - Define X^* - best possible alternative on the range
Define X_* - worst possible alternative on the range
 - Assign convenient values - $U(X^*) = 1.0$; $U(X_*) = 0.0$
 - Conduct data collection/interview to find X_i and p
Note: $U(X_i) = p$
 - Generally $p = 0.5$; 50:50 lotteries
- Repeat, substituting new X_i into lottery, as often as desired e.g. $X_2 = (X_1, 0.5; X)$



Utility Measurement - New Method

- Avoid Certainty Equivalents to Avoid “Certainty Effect”
- Consider a “Lottery Equivalent”
 - Rather than Comparing a Lottery with a Certainty
 - Reference to a Lottery is Not a Certainty



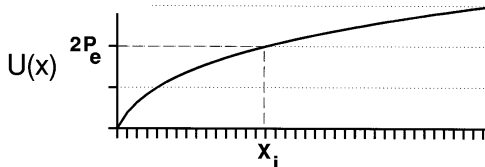
- Vary “Pe” until Indifferent between Two Lotteries. This is the “Lottery Equivalent”

Utility Measurement - New Method (contd)

- Analysis
 - $(X^*, P_e; X_*) \sim (X_i, P; X_*)$
 - $$P_e U(X^*) + (1-P_e)U(X_*) = P U(X_i) + (1-P) U(X_*)$$

$$P_e (U(X^*) - U(X_*)) = P (U(X_i) - U(X_*))$$

$$P_e = P \frac{U(X_i) - U(X_*)}{U(X^*) - U(X_*)}$$
 - $U(X_i) = P_e/P$; or $U(X_i) = 2 P_e$ when $P = 0.5$
- Graph



- Big Advantage - Avoids Large Errors (+/- 25% of “Certainty Equivalent” Method)

Example of Measurement

- **Scenario**

Your rich, eccentric relative offers you X for sure or a 50:50 chance to get _____

- **Bracketing**

if X = _____

would you take it?

would someone else?

Your indifference point is _____

Other person's is _____

- **Interpretation:**

1

U(x)

0

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Lotteries - Central to Utility Measurement

- **Uncertainty**

- Basis for Assessment of Utility
- Motivates Decision Analysis

- **Lottery - Formal Presentation of Uncertain Situation**

- **Utility Assessment -**

Compares Preferability of Alternative of Known Value with Alternative of Known Value

- **How Does One Extract Utility Information from Interview Data?**

- **How Does One Construct Lottery Basis for Interview?**

“Buying and Selling Lotteries”

- **Observable Feature of Daily Existence**
- **Obvious One Include:**
 - **Buying Lottery tickets**
 - **Gambling; Other Games of Chance**
 - **Purchase of Insurance**
- **Subtler Ones Are:**
 - **Crossing a Street against the Lights**
 - **Exceeding the Speed Limit**
 - **Illegal Street Parking**
 - **Smoking; Overeating; Drug-Taking**
- **Question: How to Analyze This Behavior?**

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Two Basic Lottery Transactions

- **Buying of Lotteries**
 - **In Absence of Transaction, Subject “Holds” an Object of Value**
 - **In Exchange for the Lottery, Subject Gives Up Valued Object**
 - **Buying “Price” Defines Net Value of Purchased Lottery**
- **Selling of Lotteries**
 - **In Absence of Transaction, Subject “Holds” a Lottery**
 - **In Exchange for the Lottery, Subject Receives a Valued Object**

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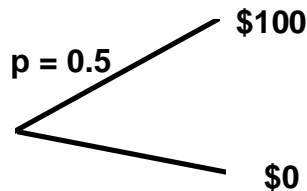
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Two Basic Lottery Transactions (contd)

- **Selling of Lotteries (contd)**
 - Selling “Price” Defines Value of Sold Lottery
- **Analytically Distinct Transactions; Must be Treated Differently**

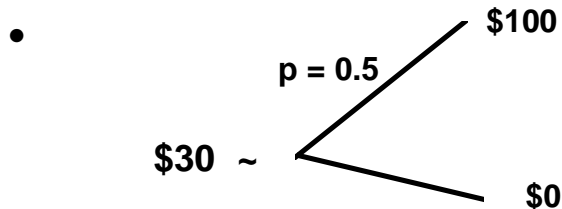
Selling Lotteries

- **Generally Easier to Understand**
- **Initially, Subject Holds a Lottery**
Example, You Own a 50:50 Chance to Win \$100



Selling Lotteries (contd)

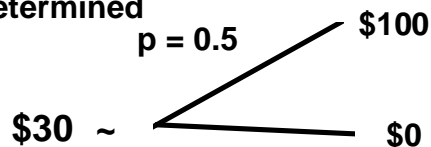
- Subject Agrees to Exchange (Sell) this Lottery for No Less Than SP = Selling Price Example: \$30



This is Called an “Indifference Statement”

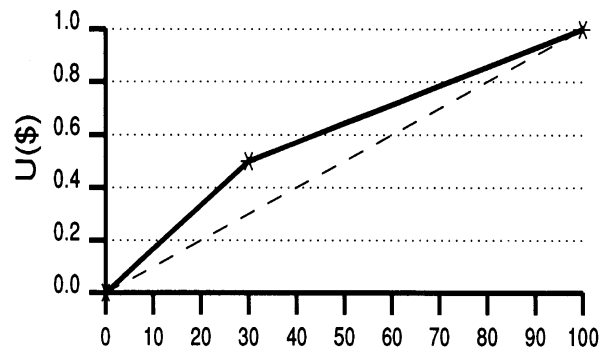
Selling Lotteries (contd)

- Based on this Indifference Statement, Utility Values can be determined



- Set $U(\$0) = 0.0$ and $U(\$100) = 1.0$.
- Translate the Indifference Statement into a Utility Statement: $U(\$30) = 0.50 U(\$0) + 0.50 U(\$100)$
- Solve for $U(\$30)$
 $U(\$30) = 0.50 (0) + 0.50 U(\$100) = 0.50$

Selling Lotteries (contd)



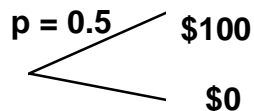
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Buying Lotteries

- The “Other” Side of the Transaction
- Subtle, but Critical Analytical Difference
- Source of Difference:
Buying Price Changes Net Effect of Lottery
- Example: Look at the Buyer in the Last Example

This Lottery was Purchased for \$30

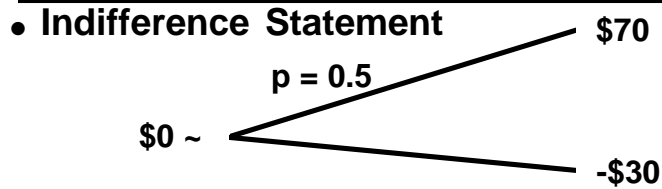


What is the Appropriate Indifference Statement?

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Buying Lotteries (contd)



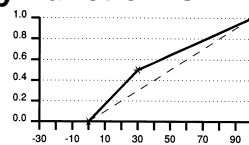
Must Explicitly Consider “Do Nothing” vs Net Outcomes

- Note:
 - Net Outcomes, Not Original Outcomes, Determine Indifference Statement
 - Set $U(-\$30) = 0$; $U(\$70) = 1$
 - $U(\$0) = 0.5 U(-\$30) + 0.5 U(\$70)$
 - $U(\$0) = 0.5$

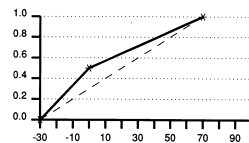
Buying Lotteries (contd)

- Resulting Utility Function is Different

➤ Seller

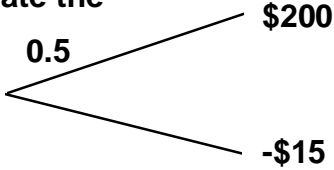
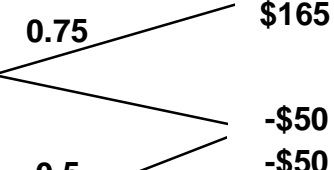



➤ Buyer



- This Should Not be Surprising. If the Utility Functions were Not Different, the Transaction would Not Have Taken Place!

Exercises: Buying and Selling Lotteries

- Given a Transaction, Generate the Indifference Statement
- ▶ Buy this Lottery for \$35
 
 - ▶ Sell this Lottery for \$50
 
 - ▶ Pay Someone \$30 to Take This Lottery
 

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Indifference Statements

Let

$$U(\$165) = 1$$

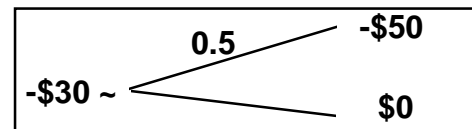
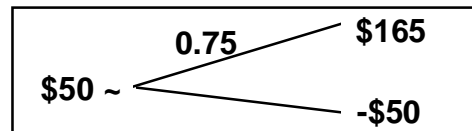
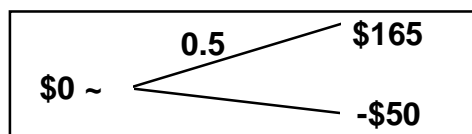
$$U(-\$50) = 0$$

Then

$$U(\$0) = 0.50$$

$$U(\$50) = 0.75$$

$$U(-\$30) = 0.25$$



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Utility Result

