Utility Assessment

- Basic Axioms
- Example
- Interview Process
- Procedures
  - Conventional
  - New
- Discussion

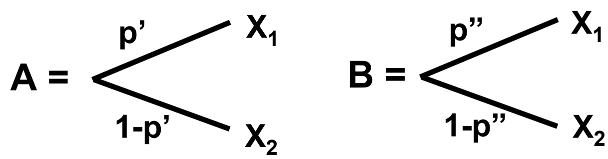
Utility Function - U(X)

- Definition:
  - U(X) is a Special V(X),
  - Defined in an Uncertain Environment

- It has a Special Advantage
  - Units of U(X) DO measure relative preferance
  - CAN be used in meaningful calculations

Basic Axioms of U(X) (1)

- Probability
  - Probabilities exist - can be quantified
  - More is better



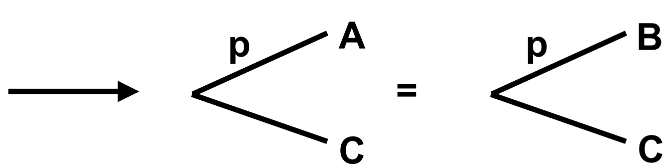
If $X_1 > X_2$; A > B if p' > p"

is preferred to

Basic Axioms of U(X) (2)

- Preferences
  - Linear in Probability

    (substitution/independence) - Equals can be substituted if a subject is indifferent between A and B



Not a good assumption for small p (high consequences) !

Cardinal Scales (1)

- **Units of interval are equal, therefore averages and arithmetic operations are meaningful**

- **Two types exist**
  - **Ratio**
    Zero value implies an absence of phenomenon
    e.g., Distance, Time
    note: F'(x) = a F(x)
    defines an equivalent measure (e.g., meters and feet)

Cardinal Scales (2)

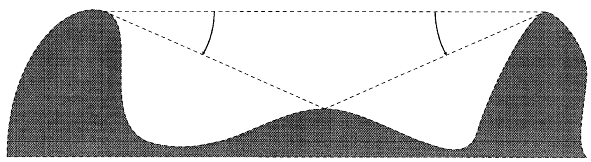
- **Ordered Metric**
  Zero is relative, arbitrary for example: Temperature

- **define two points:**
  0 degrees C - freezing point of pure water
  100 degrees C - boiling point of pure water at standard temperature and pressure
  0 degrees F - freezing point of salt water
  100 degrees F - What?

  Note: f'(x) = a f(x) + b (e.g. F = (9/5) C + 32
  equivalent measures under a positive linear transformation

Consequences of Utility Axioms

- Utility exists on an ordered metric scale
- To measure, sufficient to
  - Scale 2 points arbitrarily
  - obtain relative position of others by probability weighting -- Similar to triangulation in surveying
  - For Example: Equivalent = $(X^*, p; X_*)$



How do we Measure Utility?

- Since it is empirical -- Measure
- Since it is personal -- Measure Individuals
- Solution: Some form of Interview
  - oral
  - computer based

Interview Issues (1)

- Put person at ease
  - this individual is expert on his values
  - his opinions are valued
  - there are no wrong answers
  - THIS IS NOT A TEST!!
- Scenario relevant to
  - person
  - issues to be evaluated

Interview Issues (2)

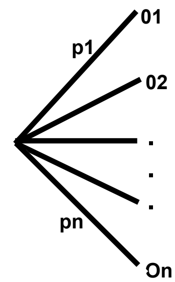
- Technique for obtaining equivalents: BRACKETING
- Basic element for measurement: LOTTERIES

Nomenclature

- **Lottery**

  A risky situation with outcomes 0_j at probability p_j

  Written as $(0_1, p_1; 0_2, p_2; \ldots)$

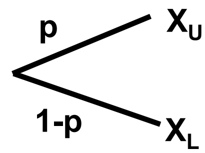


- **Binary Lottery**

  A lottery with only two branches, entirely defined by X_U, p_U, X_L

  $p(X_L) = 1 - P_U$

  Written as $(X_U, P_U; X_L)$



Nomenclature (cont'd)

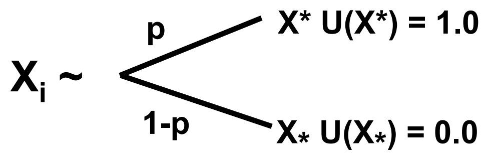
- **Elementary Lottery**

  Lottery where one outcome equals zero, that is, the status quo

written as (X,p)

Utility Measurement
Conventional Method

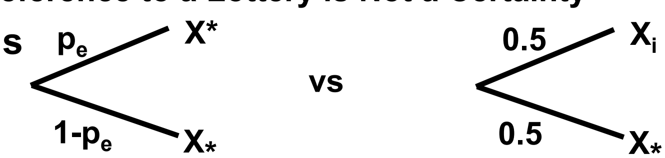
- Certainty Equivalent - Balance Xi and a lottery
  - Define X* - best possible alternative on the range
    Define X_* - worst possible alternative on the range
  - Assign convenient values - U(X*) = 1.0; U(X_*) = 0.0
  - Conduct data collection/interview to find X_i and p
    Note: $U(X_i) = p$
  - Generally p = 0.5
    50:50 lotteries



  - Repeat, substituting new Xi into lottery, as often as desired e.g. $X_2 = (X_1, 0.5; X)$

Utility Measurement
New Method (1)

- Avoid Certainty Equivalents to Avoid "Certainty Effect"
- Consider a "Lottery Equivalent"
  - Rather than Comparing a Lottery with a Certainty
  - Reference to a Lottery is Not a Certainty
- Thus



- Vary "P_e" until Indifferent between Two Lotteries. This is the "Lottery Equivalent"

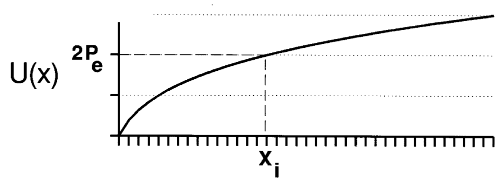
Utility Measurement New Method (2)

- Analysis $(X^*, P_e; X_*) \sim (X_i, P; X_*)$

➽ $P_e U(X^*) + (1-P_e)U(X_*) = P\, U(X_i) + (1-P)\, U(X_*)$
$P_e\, [U(X^*) - U(X_*)] = P\, [U(X_i) - U(X_*)]$
$P_e = P\, U(X_i)$

➽ $U(Xi) = P_e/P$; or $U(Xi) = 2\, P_e$ when $P = 0.5$

- Graph



- Big Advantage - Avoids Large Errors (+/- 25% of "Certainty Equivalent" Method)

Example of Measurement

- **Scenario**
  Your rich, eccentric relative offers you X for sure or a 50:50 chance to get ________
- **Bracketing**
  if X = ________
  would you take it?
  would someone else?

Your indifference point is

Other person's is

- Interpretation: 1
  U(x)
  0

Lotteries -- Central to Utility Measurement

- Uncertainty
  - Basis for Assessment of Utility
  - Motivates Decision Analysis
- Lottery - Formal Presentation of Uncertain Situation
- Utility Assessment -
  Compares Preference of Alternative of Known Value with Alternative of Known Value
- How Does One Extract Utility Information from Interview Data?
- How Does One Construct Lottery Basis for Interview?

"Buying and Selling Lotteries"

- Observable Feature of Daily Existence
- Obvious One Include:
  - Buying Lottery tickets
  - Gambling; Other Games of Chance
  - Purchase of Insurance
- Subtler Ones Are:
  - Crossing a Street against the Lights
  - Exceeding the Speed Limit
  - Illegal Street Parking
  - Smoking; Overeating; Drug-Taking
- Question: How to Analyze This Behavior?

Two Basic Lottery Transactions (1)

- **Buying of Lotteries**
  - In Absence of Transaction, Subject "Holds" an Object of Value
  - In Exchange for the Lottery, Subject Gives Up Valued Object
  - Buying "Price" Defines Net Value of Purchased Lottery

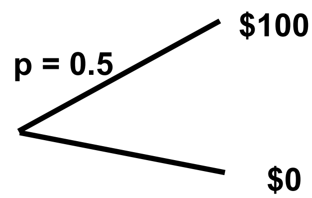
Two Basic Lottery Transactions (2)

- **Selling of Lotteries**
  - In Absence of Transaction, Subject "Holds" a Lottery
  - In Exchange for the Lottery, Subject Receives a Valued Object
  - Selling "Price" Defines Value of Sold Lottery

- **Analytically Distinct Transactions; Must be Treated Differently**

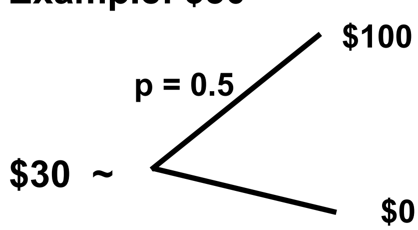
Selling Lotteries (1)

- Generally Easier to Understand
- Initially, Subject Holds a Lottery
  Example, You Own a 50:50 Chance to Win $100



Selling Lotteries (2)

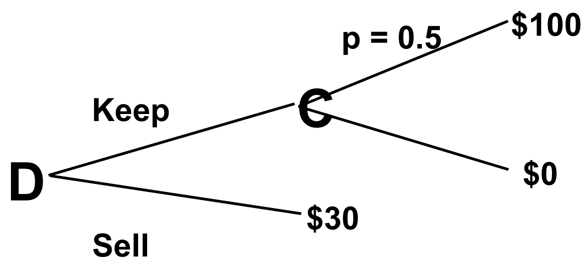
- Subject Agrees to Exchange (Sell) this Lottery for No Less Than SP = Selling Price Example: $30



This is Called an "Indifference Statement"

Selling Lotteries - Alternative View (1)

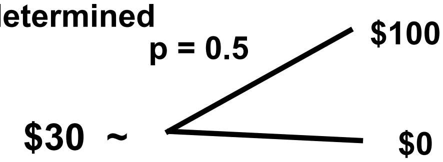
- Another way to look at lottery transactions is to express them as decision analysis situations. Selling a lottery can be represented as follows:



- When the two alternative strategies are equally valued, then we can construct an indifference statement using the two sets of outcomes.

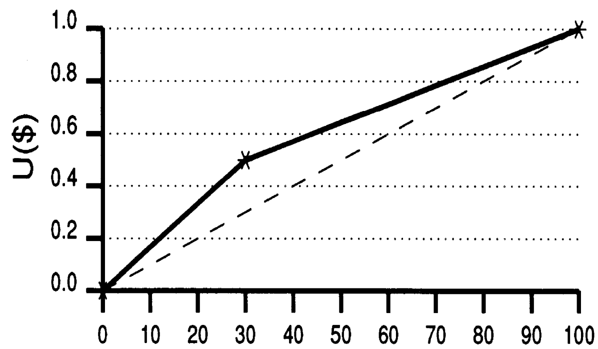
Selling Lotteries Alternative View (2)

- Based on this Indifference Statement, Utility Values can be determined



  - Set U($0) = 0.0 and U($100) = 1.0.
  - Translate the Indifference Statement into a Utility Statement: U($30) = 0.50 U($0) ≠ 0.50 U($100)
  - Solve for U($30)
    U($30) = 0.50 (0) + 0.50 U($100) = 0.50

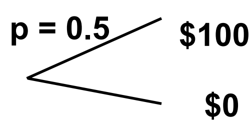
Selling Lotteries -- Graph



Buying Lotteries (1)

- The "Other" Side of the Transaction
- Subtle, but Critical Analytical Difference
- Source of Difference:
  Buying Price Changes Net Effect of Lottery
- Example: Look at the Buyer in the
  Last Example

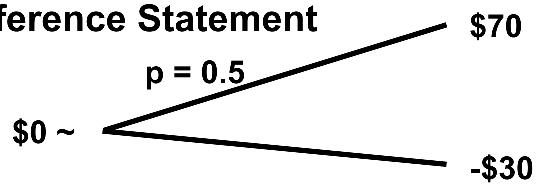
This Lottery was Purchased for $30



What is the Appropriate Indifference Statement?

Buying Lotteries (2)

- Indifference Statement

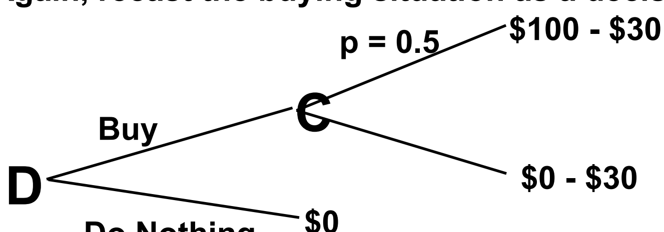


Must Explicitly Consider "Do Nothing" vs Net Outcomes

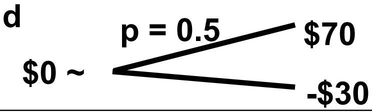
- Note:
  Net Outcomes, Not Original Outcomes, Determine Indifference Statement
  - Set U(-$30) = 0; U($70) = 1
  - U($0) = 0.5 U(-$30) + 0.5 U($70)
  - U($0) = 0.5

Buying Lotteries (3)

- Again, recast the buying situation as a decision tree



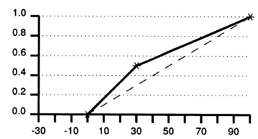
- If the buyer is just indifferent between the two decision outcomes, then the following indifference statement must hold



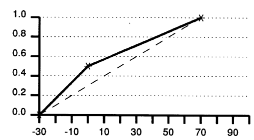
Buying Lotteries (4)

- **Resulting Utility Function is Different**

➽ **Seller**



➽ **Buyer**

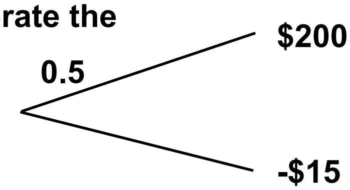


- **This Should Not be Surprising. If the Utility Functions were Not Different, the Transaction would Not Have Taken Place!**

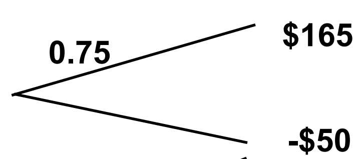
Exercises: Buying and Selling Lotteries

- **Given a Transaction, Generate the Indifference Statement**

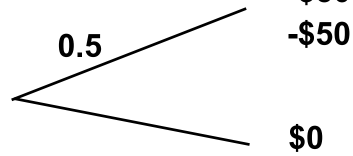
➽ **Buy this Lottery for $35**



➽ **Sell this Lottery for $50**



➽ **Pay Someone $30 to Take This Lottery**



Indifference Statements

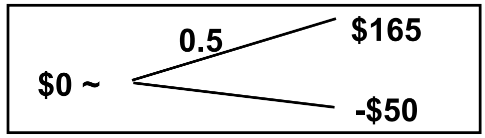
Let

U($165) = 1

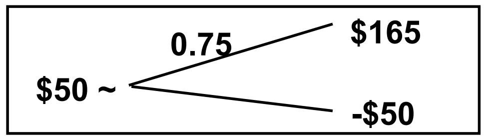
U(-$50) = 0

Then

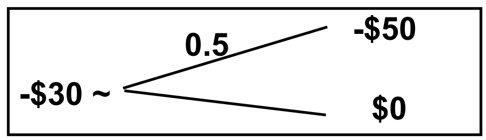
U($0) = 0.50



U($50) = 0.75



U(-$30) = 0.25



Utility Result