Multiattribute Utility

- Objective: to present a practical method to obtain U(X)
- Motivation
- Axiomatic Basis
- Procedure
- Formula

Motivation

- Curse of Dimensionality
  - Procedure for 1-dimensional utility function can, in theory, be applied to an *n*-dimensional utility function
  - But, consider the number of points to be assessed if we divide a range of N dimensions into quarters

Dimensions	Number of points
1	5 - 2 = 3
2	(5)(5) - 2 = 23
3	(5)(5)(5) - 2 = 123

Axiomatic Basis

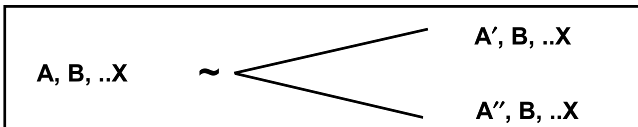
- **Preferential Independence - an ordinal condition**
  - The order of preference between any 2 pairs of outcomes is constant, regardless of level of other outcomes
  - If $(X_1', X_2') > (X_1'', X_2'')$ for any $(X_3', ... X_N')$
  - Example

    I prefer (1 cup coffee, black) to (2 cups coffee, w/ sugar), regardless of wealth
  - Consequence

    Can compare dimensions two at a time, independently of others

Automatic Basis (cont'd)

- **Utility Independence - a cardinal condition**
  - The relative intensity of value for different amounts of one type of outcome is independent of level of all other outcomes.



True for all consistent sets of B, ..X

Axiomatic Basis (cont'd)

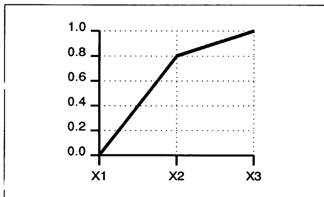
- Example
  - » When hungry, I prefer 1 plate of food for sure to a 50:50 gamble on 2 plates or none, regardless of noise.
- Consequence
  - » Can assess $U(X_i)$ once, and use it for all cross sections of U(X), subject to a positive linear transformation
  - » "Shape" of $U(X_i)$ constant

Note Concerning "Constancy of Shape"

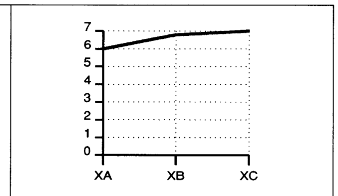
- To say that a utility function retains its "shape" means that the utility function can only undergo a constant linear transformation, i.e. $U'(\underline{X}) = a\, U(\underline{X}) + b$

Note Concerning "Constancy of Shape" (cont'd)

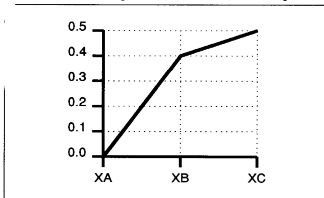
Examples



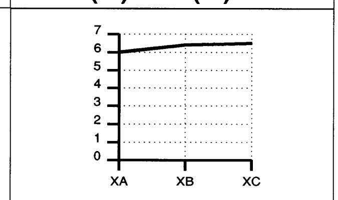
Xb ~ (Xc, 0.8, Xa)



U'(X) = U(X) + 6



U'(X) = 0.5 U(X)



U'(X) = 0.5 U(X) + 6

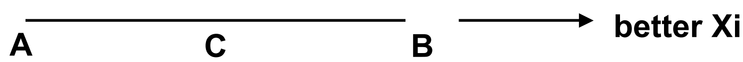
Procedure for U(X)

- Establish the range of each dimension - X_i^* to X_{i*}
- Set $X_* = (X_{1*}, ..., X_{n*})$ $U(X_*) = 0$
  $X^* =$ (all the best) $U(X^*) = 1$
- Establish the relative value of each dimension:

$(X_{*1}, ...X_i^*,...X_{*n})$ ~ k_i X all best ; X all worst

Procedure for U(X) (cont'd)

- Estimate 1-dimensional $U(X_i)$; scale from 0 -> 1 for each case
- Scale 1-dimensional $U(X_i)$ into U(X) Between any two points in X

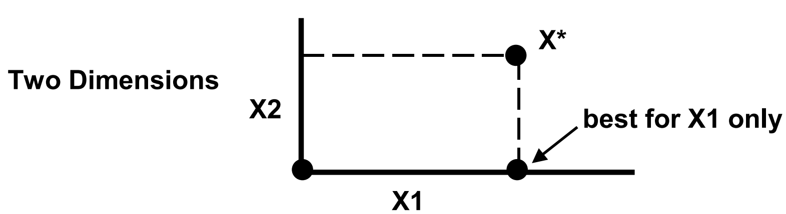


UC = UA + p(UB - UC)
p = proportion from $U(X_i)$

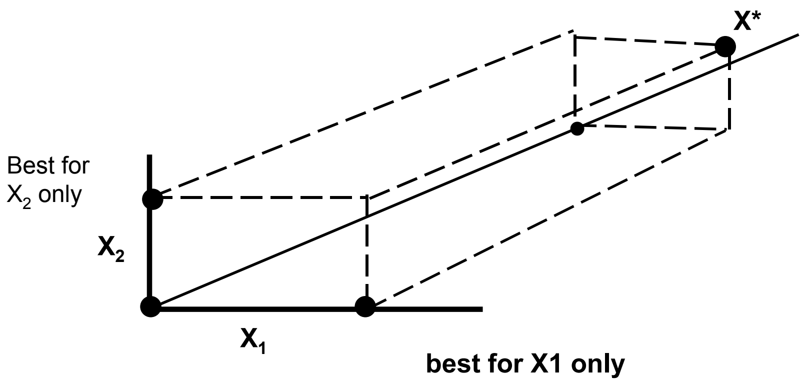
To Establish Relative Value of Each Dimension

- Balancing Act between "best" and "worst" over all X
- For best in one dimension only

Graphically



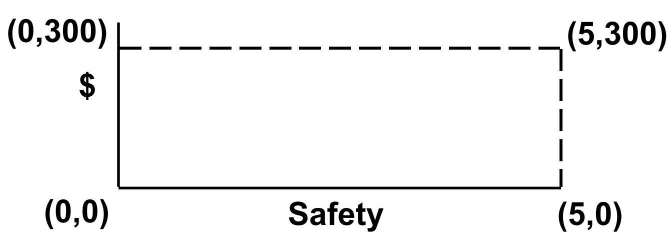
To Establish Relative Value of Each Dimension



MAUA Example

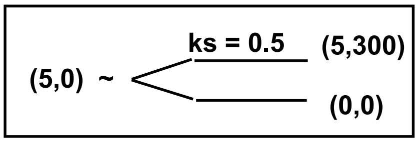
U(X) X1 = Safety X2 = Profit

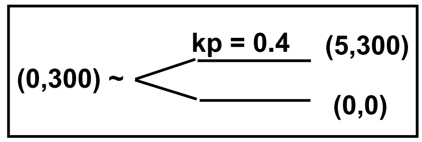
1. $X_{1*} = 0$; $X_1^* = 5$
   $X_{2*} = 0$; $X_2^* = 300$
2. U(0,0) = 0; U(5,300) = 1



MAUA Example (cont'd)

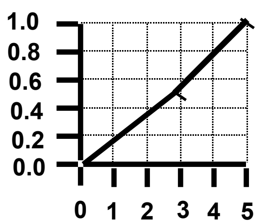
3.

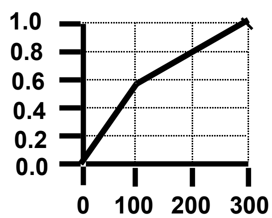


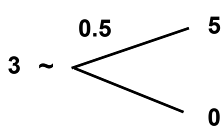


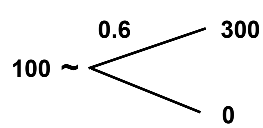
Example (cont'd)

4. Single attribute functions



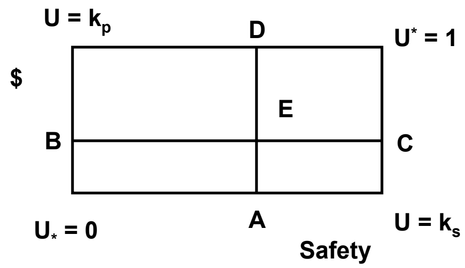






Example (cont'd)

5. **Assessments**



UA = 0 + 1/2 (0.5 - 0) = 0.25
UB = 0 + 0.6 (0.4 - 0) = 0.24
UD = 0.4 + 1/2 (1 - 0.4) = 0.70
UC = 0.5 + 0.6 (1 - 0.5) = 0.80
UE = 0.25 + 0.6 (0.45) = 0.52
= 0.24 + 1/2 (0.56) = 0.52

Formula

$$K\,U(\underline{X}) + 1 = \prod_i (K\,K_i\,U_i(X) + 1)$$

U(X) and U(X_i) all scaled between 0 and 1

- **For 2 dimensions, quadratic solutions make it possible to solve directly for** K
- $K = (1 - K_1 - K_2)/K_1K_2$
- **For larger numbers of dimensions, iterative solutions (e.g., Newton's method) appropriate**

Formula (cont'd)

$$K\,U(\underline{X}) + 1 = \prod_i (K\ K_i\,U_i(X) + 1)$$

- Guidelines:

  If the sum of all ki < 1 $\quad K > 0$

  If the sum of all ki > 1 $\quad -1 < K < 0$

  If the sum of all ki = 1 $\quad K = 0$;

  $U(X) = \sum K_i\,U_i(X_i)$