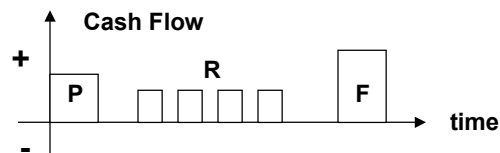


## Present Value, Discounted Cash Flow. Engineering Economy

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- **Objective:**
  - To provide economic comparison of benefits and costs that occur over time
- **Assumptions:**
  - All Benefits, Costs measured in money
  - Single point of view



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## Issue - Value over time

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- Money now has a different value than the same amount at a different date
- Comparable to }  
not equal to } interest rate
- Proper name: Discount Rate,  $r$   
because future benefits/costs are reduced (that is, “discounted”) to compare with present

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## Formulas for N Periods

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- **Single amounts**

a) **Future Amount** =  $P (1 + r)^N = P (\text{caf})$

**caf** = Compound Amount Factor

b) **Present Amount** =  $F/\text{caf}$

$1/\text{caf}$  = Present Worth Factor

- **Finite Series**

c)  $F = \sum_i R (1 + r)^i = R [(1 + r)^N - 1] / r$

d)  $R = P (\text{crf}) = [P * r (1 + r)^N] / [(1 + r)^N - 1]$

**crf** = Capital Recovery Factor

## Formulas for N Periods (cont')

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- **Infinite Series**

$1 \ll (1 + r)^N \Rightarrow (1 + r)^N / [(1 + r)^N - 1] \rightarrow 1 \Rightarrow \text{crf} \rightarrow r$

- **Small Periods**

$(1 + r)^N \Rightarrow e^{rN}$

## Discount Rate Approximation

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- **To appreciate effect of discounting:**  
 “Rule of 72” or “Rule of 70”  
 $e^{rN} = 2.0$  when  $rN = 0.72$  (actually = 0.693)
  
- **Therefore, present amount doubles when future amount halves**  
 $rN = 72$  with  $r$  expressed in percent
  
- **Examples**
  - When would \$1000 invested at 10% double?
  - What is, at 9%, the value of \$1000 in 8 years?

## Example Application of Present Value Analysis

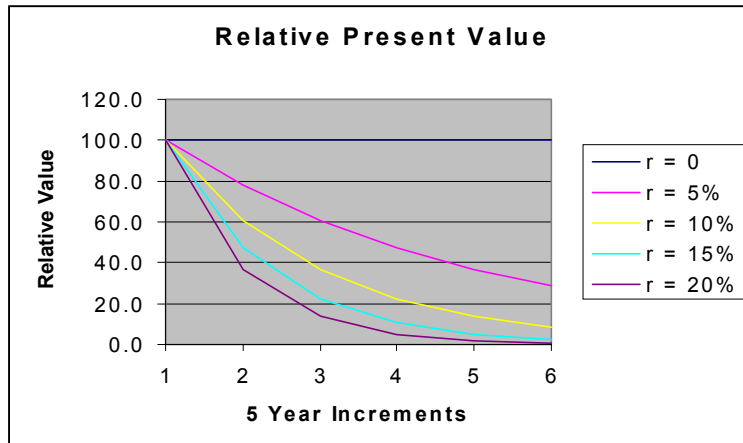
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**All by spreadsheets!**

**Example:**

Year	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
Investment	15			3			5			
Net Income		2	3	4	5	5	3	4	5	6
Cash Flow	-15	2	3	1	5	5	-2	4	5	6
NPV at 12%	\$0.79		Formula: NPV(12%, B9:K9)							

## Graphical view of Effect of Different Discount Rates and Lengths of Time



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## Effect of Different Discount Rates

- **Higher Discount rates =>**
  - smaller value of future benefits
  - discourages projects with long pay back periods
  - project advocates try to minimize discount rate
  - Examples: Massive Dams -- 3 Gorges in China
- **Argument over Discount rates**
  - Often very difficult politically
  - Not hard from technical perspective
  - Focus on Opportunity Cost
  - See US Government Position (next)
  - Is 7% real (net of inflation) correct ??

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## Current position of US Government on Discount rate (OMB Circular A-94)

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### 1. Base-Case Analysis.

Constant-dollar benefit-cost analyses of proposed investments and regulations should report net present value and other outcomes determined using a real discount rate of 7 percent. This rate approximates the marginal pretax rate of return on an average investment in the private sector in recent years. Significant changes in this rate will be reflected in future updates of this Circular.

### 2. Other Discount Rates.

Analyses should show the sensitivity of the discounted net present value and other outcomes to variations in the discount rate. The importance of these alternative calculations will depend on the specific economic characteristics of the program under analysis. For example, in analyzing a regulatory proposal whose main cost is to reduce business investment, net present value should also be calculated using a higher discount rate than 7 percent.

## Effect of Different Time Horizons

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### • Longer Periods of Benefits

- Increase Present Values
- Increment depends on discount rate

### • What length of time matters?

- For US Government Rate, not much over 30 years
- For Rates commonly used in business (15 to 20%), anything over 20 years has little value
- Exception: Future benefits grow exponentially

## Summary

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- **Formulas Simple**
- **Especially by Spreadsheets**
  
- **Discount rate is key issue**
- **High rates recommended (but see later presentations)**
- **Longer term benefits not large**