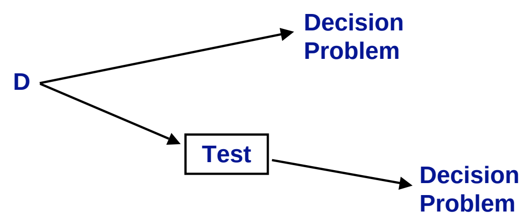
Value of Information

Information Collection - Key Strategy

- **Motivation**
  - To reduce uncertainty which makes us choose "second best" solutions as insurance
- **Concept**
  - Insert an information-gathering stage (e.g., a test) before decision problems, as an option



Operation of Test

New Information
↓
Revision of Prior Probabilities in Decision Problem
↓
New Expected Values in Decision Problem

EV (after test) $\geq$ EV (without test)

- **Why?**
  - Because we can avoid bad choices and take advantage of good ones, in light of test results
- **Question:**
  - Since test generally has a cost, is the test worthwhile?

    What is the value of information?
    Does it exceed the cost of the test?

Value of Information - Essential Concept

- **Value of information is an expected value**
- **Expected value after test "k"**

$$= \sum_k p_k(D_k^*)$$

Pk = probablility, after test k, of an observation which will lead to an optimal decision (incorporating revised probabilities due to observation) D_k^*

- **Expected Value of information**

= EV (after test) - EV (without test)

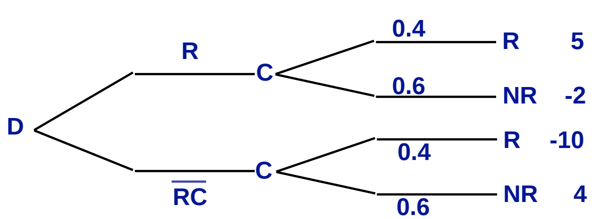
$$= \sum_k p_k(D_k^*) - \sum_k p_k(E_j)O_{ij}$$

Expected Value of Perfect Information - EVPI

- Perfect information is a hypothetical concept
- Use: Establishes an upper bound on value of any test
- Concept: Imagine a "perfect" test which indicated exactly which Event, E_j, will occur
  - By definition, this is the "best" possible information
  - Therefore, the "best" possible decisions can be made
  - Therefore, the EV gain over the "no test" EV must be the maximum possible - an upper limit on the value of any test!

EVPI Example

- Question: Should I wear a raincoat?
  RC - Raincoat; $\overline{RC}$ - No Raincoat
- Two possible Uncertain Outcomes
  (p = 0.4) or No Rain (p = 0.6)



- Remember that better choice is to take raincoat, EV = 0.8

EVPI Example (continued)

- Perfect test

- EVPI

EV (after test)	=	0.4(5) + 0.6(4)	=	4.4
EVPI =	4.4 - 0.8 = 3.6			

Application of EVPI

- A major advantage: EVPI is simple to calculate
- Notice:
  - Prior probability of the occurrence of the uncertain event must be equal to the probability of observing the associated perfect test result
  - As a "perfect test", the posterior probabilities of the uncertain events are either 1 ot 0
  - Optimal choice generally obvious, once we "know" what will happen
- Therefore, EVPI can generally be written directly
- No need to use Bayes' Theorem

Expected Value of Sample Information - EVSI

- Sample information are results taken from an actual test $0 \leq EVSI \leq EVPI$
- Calculations required
  - Obtain probabilities of test results, p_k
  - Revise prior probabilities p_j for each test result $TR_k \Rightarrow p_{jk}$
  - Calculate best decision D_k^* for each test result TRk (a k-fold repetition of the original decsion problem)
  - Calculate EV (after test) = $\sum_k p_k(D_k^*)$
  - Calculate EVSI as the difference between EV (after test) - EV (without test)
- A BIG JOB

EVSI Example

- Test consists of listening to forecasts
- Two possible test results
  - Rain predicted = RP
  - Rain not predicted = NRP
- Assume the probability of a correct forecast = 0.7

  p(RP/R) = P(NRP/NR) = 0.7

  P(NRP/R) = P(RP/NR) = 0.3
- First calculation: probabilities of test results

P(RP) = p(RP/R) p(R) + P(RP/NR) p(NR)
= (0.7) (0.4) + (0.3) (0.6) = 0.46

P(NRP) = 1.00 - 0.46 = 0.54

EVSI Example (continued 2 of 5)

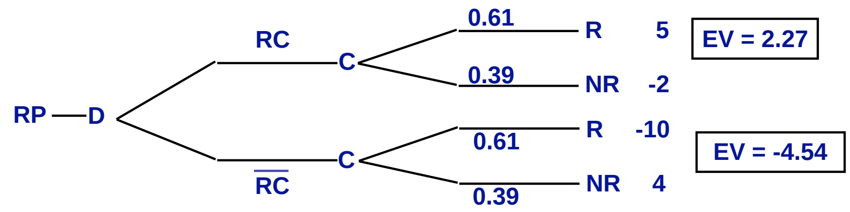
- **Next: Posterior Probabilities**

P(R/RP) = p(R) (p(RP/R)/p(RP)) = 0.4(0.7/0.46) = 0.61
P(NR/NRP) = 0.6(0.7/0.54) = 0.78

Therefore, p(NR/RP) = 0.39 & p(R/RNP) = 0.22

EVSI Example (continued 3 of 5)

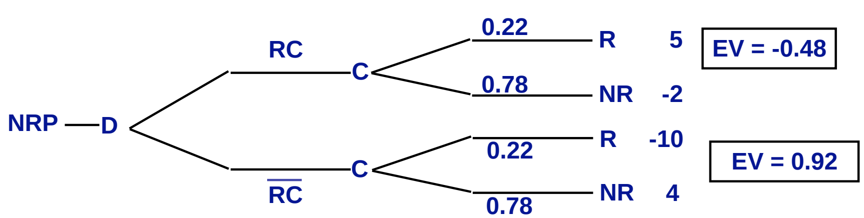
- **Best decisions conditional upon test results**



EV (RC) = (0.61) (5) + (0.39) (-2) = 2.27
EV (RC) = (0.61) (-10) + (0.39) (4) = -4.54

EVSI Example (continued 4 of 5)

- **Best decisions conditional upon test results**



EV (RC) = (0.22) (5) + (0.78) (-2) = -0.48
EV (RC) = (0.22) (-10) + (0.78) (4) = 0.92

EVSI Example (continued 5 of 5)

- **EV (after test)**
  = p(rain pred) (EV(strategy/RP))
  + P(no rain pred) (EV(strategy/NRP))
  = 0.46 (2.27) + 0.54 (0.92) = 1.54
- **EVSI = 1.54 - 0.8 = 0.74 < EVPI**

Practical Example - Is a Test Worthwhile?

- **If value is Linear (i.e., probabilistic expectations correctly represent valuation of outcomes under uncertainty)**
  - **Calculate EVPI**
  - **If EVPI < cost of test → Reject test**
  - **Pragmatic rule of thumb**
    If cost > 50% EVPI → Reject test
    (Real test are not close to perfect)
  - **Calculate EVSI**
  - **EVSI < cost of test → Reject test**
  - **Otherwise, accept test**

Is Test Worthwhile? (continued)

- **If Value Non-Linear (i.e., probabilistic expectation of value of outcomes does NOT reflect attitudes about uncertainty)**
- **Theoretically, cost of test should be deducted from EACH outcome that follows a test**
  - **If cost of test is known**
    A) Deduct costs
    B) Calculate EVPI and EVSI (cost deducted)
    C) Proceed as for linear EXCEPT
    Question is if EVPI(cd) or EVSI(cd) > 0?
  - **If cost of test is not known**
    A) Iterative, approximate pragmatic approach must be used
    B) Focus first on EVPI
    C) Use this to estimate maximum cost of a test