

## Dynamic Strategic Planning

### Risk Assessment

## Risk Assessment

- The quantified description of the uncertainty concerning situations and outcomes
- Objective: To present
  - The problem
  - Means of assessment
  - Useful formulas
  - Biases in assessment

## Methods of Assessment

- Logic
  - Example: Prob (Queen) in a deck of cards
- Frequency
  - Example:  
Prob (failure of dams) = 0.00001/dam/year

## Methods of Assessment (cont'd)

- Statistical Models
  - Example: Future Demand =  $f(\text{variables}) + \text{error}$
- Judgment
  - “Expert Opinion”
  - “Subjective Probability”
  - Example:  
Performance in 10 years of a new technology  
Major War in the Middle East

## Importance Biases in Subjective Probability Assessments

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- **Overconfidence**
  - Distribution typically much broader than we imagine
- **Insensitivity to New Information**
  - Information typically should cause us to change opinions more than it does

## Revision of Estimates (I) - Bayes Theorem

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- **Definitions**
  - $P(E)$  Prior Probability of Event  $E$
  - $P(E/O)$  Posterior  $P(E)$ , after observation  $O$  is made. This is the goal of the analysis.
  - $P(O/E)$  Conditional probability that  $O$  is associated with  $E$
  - $P(O)$  Probability of Event (Observation)  $O$
- **Theorem:  $P(E/O) = P(E) \{P(O/E) / P(O)\}$**
- **Note: Importance of revision depends on:**
  - rarity of observation  $O$
  - extremes of  $P(O/E)$

## Application of Bayes Theorem

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- **At a certain educational establishment:**
  - $P(\text{students}) = 2/3$        $P(\text{staff}) = 1/3$
  - $P(\text{fem}/\text{students}) = 1/4$        $P(\text{fem}/\text{staff}) = 1/2$
- **What is the probability that a woman on campus is a student?**

## Application of Bayes Theorem

- At a certain educational establishment:

$$P(\text{students}) = 2/3 \quad P(\text{staff}) = 1/3$$

$$P(\text{fem}/\text{students}) = 1/4 \quad P(\text{fem}/\text{staff}) = 1/2$$

- What is the probability that a woman on campus is a student?

{i.e., what is  $P(\text{student}/\text{fem})$ ??}

$$P(\text{student}/\text{fem}) = P(\text{student}) \frac{P(\text{fem}/\text{student})}{P(\text{fem})}$$

- Therefore:  $P(\text{student}/\text{fem}) = 2/3 \{(1/4) / (1/3)\}$   
 $= 1/2$

## Revision of Estimates (II) - Likelihood Ratios

- Definitions

$$P(\bar{E}) = P(E \text{ does not occur})$$

$$= > P(E) + P(\bar{E}) = 1.0$$

$$LR = P(E)/P(\bar{E}); \text{ therefore}$$

$$PE = LR / (1 + LR)$$

$$LR_i = LR \text{ after } i \text{ observations}$$

## Revision of Estimates (II) - Likelihood Ratios (cont'd)

- Formula

$$LR_1 = \frac{P(E) \{P(O_j/E) / P(O_j)\}}{P(\bar{E}) \{P(O_j/\bar{E}) / P(O_j)\}}$$

$$CLR_i = P(O_j/E) / P(O_j/\bar{E})$$

$$LR_n = LR_0 \prod_j (CLR_j)^{n_j}$$

$n_j$  = number of observations of type  $O_j$

## Application of Likelihood Ratios

- Bottle-making machines can be either OK or defective. 10% probability of it being defective.
- The frequency of cracked bottles depends upon the state of the machine. If the machine is defective, the probability of getting a cracked bottle is 20%. If the machine is OK, the probability of getting a cracked bottle is only 5%.
- Picking up 5 bottles at random from a machine, we find 2 cracked, 3 uncracked. What is the probability that the machine is defective?

## Application of Likelihood Ratios

- Bottle-making machines can be either OK or defective  $P(D) = 0.1$
- The frequency of cracked bottles depends upon the state of the machine

$$P(C/D) = 0.2$$

$$P(C/OK) = 0.05$$

## Application of Likelihood Ratios (cont'd)

- Picking up 5 bottles at random from a machine, we find {2 cracked, 3 uncracked}.  
What is the Prob(machine defective)

$$LR_o = P(D) / P(OK) = 0.1/0.9 = 1/9$$

$$CLR_c = 0.2/0.05 = 4$$

$$CLR_{uc} = 0.8/0.95 = 16/19$$

$$LR_5 = (1/9) (4)^2 (16/19)^3$$

$$P(D/\{2C, 3UC\}) = 0.52$$