## Dynamic Strategic Planning

### Utility Analysis



## Value Functions

- **In General:**

  **Preference Measure**

  **PM = f(X)**

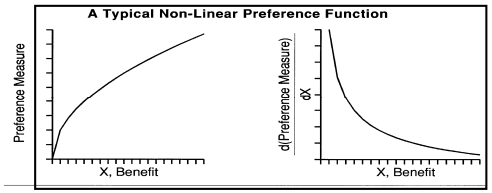
  **where X = vector of attributes**

- **Semantic Caution: Value**
  - **Value in Exchange**
  - **Value in Use**
  - **"Fair Market Value"**



## Value Function - V(X)

- **Definition:**
  - **V(X) is a means of ranking the relative preference of an individual for a bundle on consequences, X**






## Basic Axioms of Value Functions - V(X)

- **Completeness or Complete Preorder**
  - **People have preferences over all $X_i$**
- **Transitivity**
  - **If $X_1$ is preferred to $X_2$; and $X_2$ is preferred to $X_3$; Then $X_1$ is preferred to $X_3$**
  - **Caution: Assumed True for Individuals; NOT Groups**

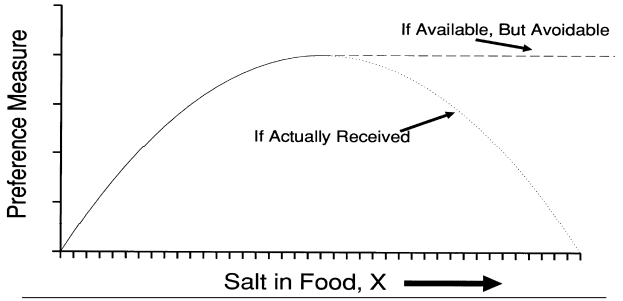
## Basic Axioms of Value Functions - V(X) (cont'd)

- **Monotonicity or Archimedean Principle**
  - **For any $\underline{X}_i$ ($\underline{X}^* \geq \underline{X}_i \square \underline{X}_*$) there is a w (0 < w < 1) such that $V(\underline{X}_i) = w\, V(\underline{X}^*) + (1 - w)\, V(\underline{X}_*)$**
  - **That is, More is Better (or Worse)**



## Another Preference Function

- **Represents a Benefit Which Ultimately Becomes Undesirable**






## Consequence of V(X) Axioms

- **Existence of $V(\underline{X})$**
- **Ranking Only**

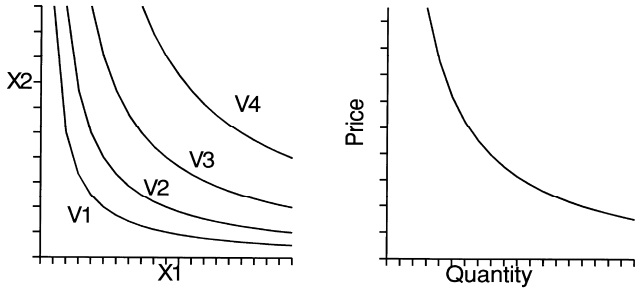
**Strategic Equivalence of Many Forms of $V(\underline{X})$**

**Any Monotonic Transform of a V(X) is Still an Equivalent $V(\underline{X})$**

$$\text{e.g., } V(X_1, X_2) = X_1^2 X_2 = 2 \log (X_1) + \log (X_2)$$



## Value Functions



## Utility Function - U(X)

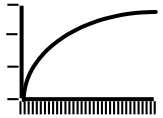
- **Definition:**
  - U(X) Is A Special V(X), Defined In An Uncertain Environment
- **It Has A Special Advantage**
  - Units of U(X) Do Measure Relative Preference
  - Can Be Used In Meaningful Calculations



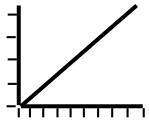
## Some Conventional Definitions

Unfortunately, Terms Can Be Misleading)

- **Risk Aversion**
  Risk Averse if X is Preferred to an Uncertain Event Whose Expected Value is X



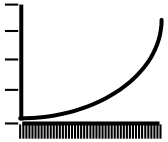
- **Risk Neutral**



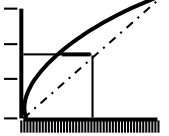


## Some Conventional Definitions (cont'd)

- **Risk Positive (Preference, or Prone!)**

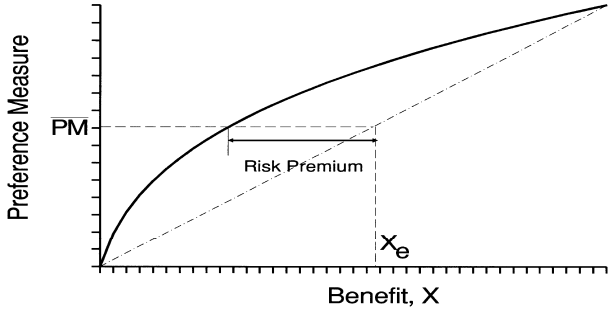


- **Risk Premium**





## Representation of "Risk Averse" Behavior



## Utility Assessment

- Basic Axioms
- Example
- Interview Process
- Procedures
  - Conventional
  - New
- Discussion



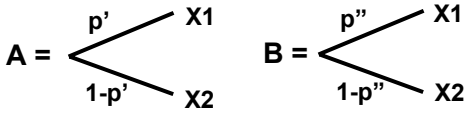
## Utility Function - U(X)

- Definition:
  - U(X) is a Special V(X),
  - Defined in an Uncertain Environment
- It has a Special Advantage
  - Units of U(X) DO measure relative preferance
  - CAN be used in meaningful calculations



## Basic Axioms of U(x)

- Probability
  - Probabilities exist - can be quantified
  - More is better



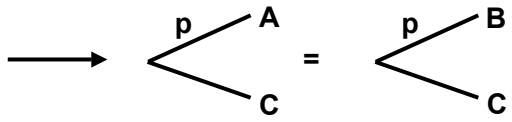

**If X1 > X2; A > B if p' > p"**



## Basic Axioms of U(x) (cont'd)

- Preferences
  - Linear in Probability

    (substitution/independence) - Equals can be substituted if a subject is indifferent between A and B




**Not a good assumption for small p (high consequences) !**

## Cardinal Scales

- Units of interval are equal, therefore averages and arithmetic operations are meaningful
- Two types exist
  - Ratio
    Zero value implies an absence of phenomenon
    e.g., Distance, Time
    note: F'(x) = a F(x)
    defines an equivalent measure (e.g., meters and feet)



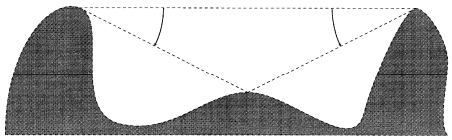
## Cardinal Scales (cont'd)

- Ordered Metric
  Zero is relative, arbitrary
  e.g., Temperature
  define two points:
  0 degrees C - freezing point of pure water
  100 degrees C - boiling point of pure water at standard temperature and pressure
  0 degrees F - freezing point of salt water
  100 degrees F - What?
  Note: f'(x) = a f(x) + b (e.g. F = (9/5) C + 32
  equivalent measures under a positive linear transformation



## Consequences of Utility Axioms

- Utility exists on an ordered metric scale
- To measure, sufficient to
  - Scale 2 points arbitrarily
  - obtain relative position of others by probability weighting
    e.g., Equivalent = $(X^*, p; X_*)$
    Similar to triangulation in surveying





## Interview Issues

- Put person at ease
  - this individual is expert on his values
  - his opinions are valued
  - there are no wrong answers
  - THIS IS NOT A TEST!!
- Scenario relevant to
  - person
  - issues to be evaluated

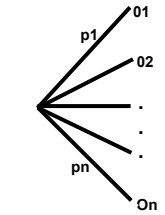
## Interview Issues (cont'd)

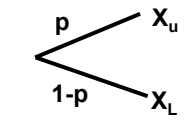
- Technique for obtaining equivalents: BRACKETING
- Basic element for measurement: LOTTERIES



## Nomenclature

- Lottery
  A risky situation with outcomes $0_j$
  at probability $p_j$
  Written as ($0_1$, $p_1$; $0_2$, $p_2$; ...)
- Binary Lottery
  A lottery with only two branches,
  entirely defined by $X_u$, $p_u$, $X_L$
  $p(X_L) = 1 - P_u$
  Written as ($X_u$, $P_u$; $X_L$)





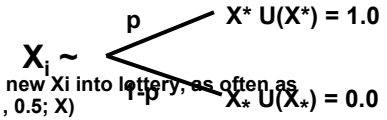

## Nomenclature (cont'd)

- Elementary Lottery
  Lottery where one outcome equals zero, i.e. status quo written as (X,p)



## Utility Measurement

- Conventional Method
- Certainty Equivalent - Balance Xi and a lottery
  - Define X* - best possible alternative on the range
    Define $X_*$ - worst possible alternative on the range
  - Assign convenient values - $U(X^*) = 1.0$; $U(X_*) = 0.0$
  - Conduct data collection/interview to find Xi and p
    Note: U(Xi) = p
  - Generally p = 0.5;
    50:50 lotteries
  - Repeat, substituting new Xi into lottery, as often as desired e.g. X2 = (X1, 0.5; X)



## Utility Measurement - New Method

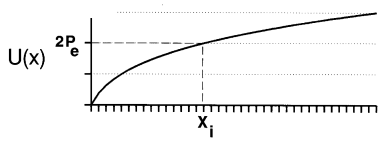
- Avoid Certainty Equivalents to Avoid "Certainty Effect"
- Consider a "Lottery Equivalent"
  - Rather than Comparing a Lottery with a Certainty
  - Reference to a Lottery is Not a Certainty
- Thus


- Vary "Pe" until Indifferent between Two Lotteries. This is the "Lottery Equivalent"



## Utility Measurement - New Method (contd)

- Analysis
  $(X^*, Pe; X_*) \sim (Xi, P; X_*)$
- ➠ $PeU(X^*) + (1-Pe)U(X_*) = P\,U(Xi) + (1-P)\,U(X_*)$
  $Pe\,(U(X^*) - U(X_*)) = P\,(U(Xi) - U(X_*))$
  $Pe = P\,U(Xi)$
- ➠ $U(Xi) = Pe/P$; or $U(Xi) = 2\,Pe$ when $P = 0.5$
- Graph




- Big Advantage - Avoids Large Errors (+/- 25% of "Certainty Equivalent" Method)





## Lotteries - Central to Utility Measurement

- Uncertainty
  - Basis for Assessment of Utility
  - Motivates Decision Analysis
- Lottery - Formal Presentation of Uncertain Situation
- Utility Assessment - Compares Preferability of Alternative of Known Value with Alternative of Known Value
- How Does One Extract Utility Information from Interview Data?
- How Does One Construct Lottery Basis for Interview?

## "Buying and Selling Lotteries"

- Observable Feature of Daily Existence
- Obvious One Include:
  - Buying Lottery tickets
  - Gambling; Other Games of Chance
  - Purchase of Insurance
- Subtler Ones Are:
  - Crossing a Street against the Lights
  - Exceeding the Speed Limit
  - Illegal Street Parking
  - Smoking; Overeating; Drug-Taking
- Question: How to Analyze This Behavior?



## Two Basic Lottery Transactions

- Buying of Lotteries
  - In Absence of Transaction, Subject "Holds" an Object of Value
  - In Exchange for the Lottery, Subject Gives Up Valued Object
  - Buying "Price" Defines Net Value of Purchased Lottery
- Selling of Lotteries
  - In Absence of Transaction, Subject "Holds" a Lottery
  - In Exchange for the Lottery, Subject Receives a Valued Object



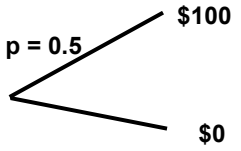
## Two Basic Lottery Transactions (contd)

- Selling of Lotteries (contd)
  - Selling "Price" Defines Value of Sold Lottery
- Analytically Distinct Transactions; Must be Treated Differently



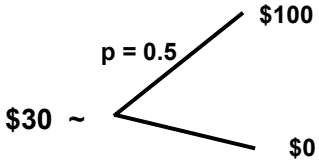
## Selling Lotteries

- Generally Easier to Understand
- Initially, Subject Holds a Lottery Example, You Own a 50:50 Chance to Win $100



## Selling Lotteries (contd)

- Subject Agrees to Exchange (Sell) this Lottery for No Less Than SP = Selling Price Example: $30

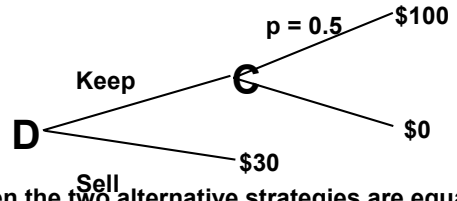



This is Called an "Indifference Statement"



## Selling Lotteries - Alternative View

- Another way to look at lottery transactions is to express them as decision analysis situations. Selling a lottery can be represented as follows:

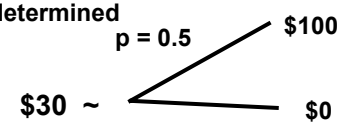



- When the two alternative strategies are equally valued, then we can construct an indifference statement using the two sets of outcomes.



## Selling Lotteries (contd)

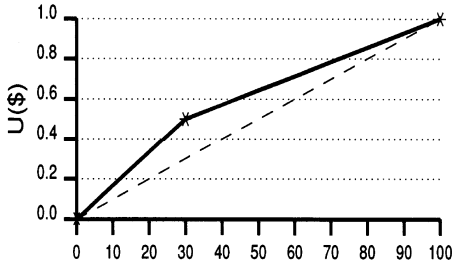
- Based on this Indifference Statement, Utility Values can be determined




  - Set U($0) = 0.0 and U($100) = 1.0.
  - Translate the Indifference Statement into a Utility Statement: U($30) = 0.50 U($0) + 0.50 U($100)
  - Solve for U($30)
    U($30) = 0.50 (0) + 0.50 U($100) = 0.50



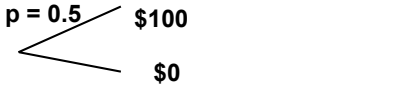
## Selling Lotteries (contd)



## Buying Lotteries

- The "Other" Side of the Transaction
- Subtle, but Critical Analytical Difference
- Source of Difference:
  Buying Price Changes Net Effect of Lottery
- Example: Look at the Buyer in the Last Example

  This Lottery was Purchased for $30

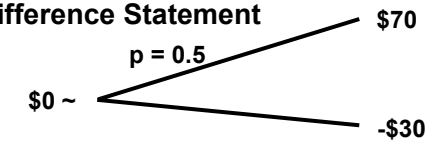



What is the Appropriate Indifference Statement?



## Buying Lotteries (contd)

- Indifference Statement



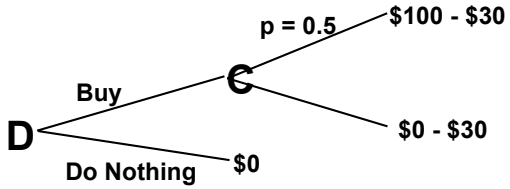

Must Explicitly Consider "Do Nothing" vs Net Outcomes

- Note:
  Net Outcomes, Not Original Outcomes, Determine Indifference Statement
  - Set U(-$30) = 0; U($70) = 1
  - U($0) = 0.5 U(-$30) + 0.5 U($70)
  - U($0) = 0.5

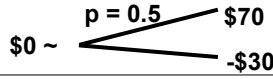


## Buying Lotteries (contd)

- Again, recast the buying situation as a decision tree




- If the buyer is just indifferent between the two decision outcomes, then the following indifference statement must hold

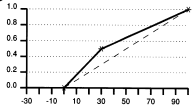




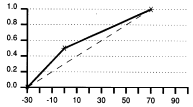

## Buying Lotteries (contd)

- Resulting Utility Function is Different

➳ Seller




➳Buyer

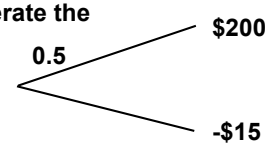



- This Should Not be Surprising. If the Utility Functions were Not Different, the Transaction would Not Have Taken Place!

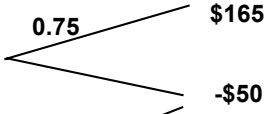
## Exercises: Buying and Selling Lotteries

- Given a Transaction, Generate the Indifference Statement

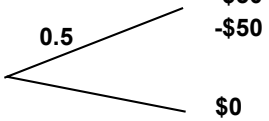
➳ Buy this Lottery for $35




➳ Sell this Lottery for $50



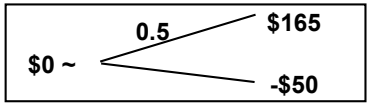

➳ Pay Someone $30 to Take This Lottery



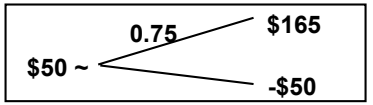



## Indifference Statements

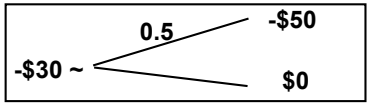
Let
U($165) = 1
U(-$50) = 0
Then
U($0) = 0.50




U($50) = 0.75




U(-$30) = 0.25






## Utility Result