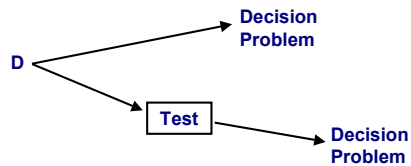
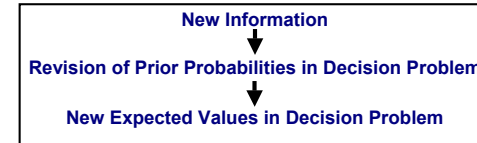


## Information Collection - Key Strategy

- **Motivation**
  - To reduce uncertainty which makes us choose “second best” solutions as insurance
- **Concept**
  - Insert an information-gathering stage (e.g., a test) before decision problems, as an option



## Operation of Test



$EV(\text{after test}) \geq EV(\text{without test})$

- **Why?**
  - Because we can avoid bad choices and take advantage of good ones, in light of test results
- **Question:**
  - Since test generally has a cost, is the test worthwhile?

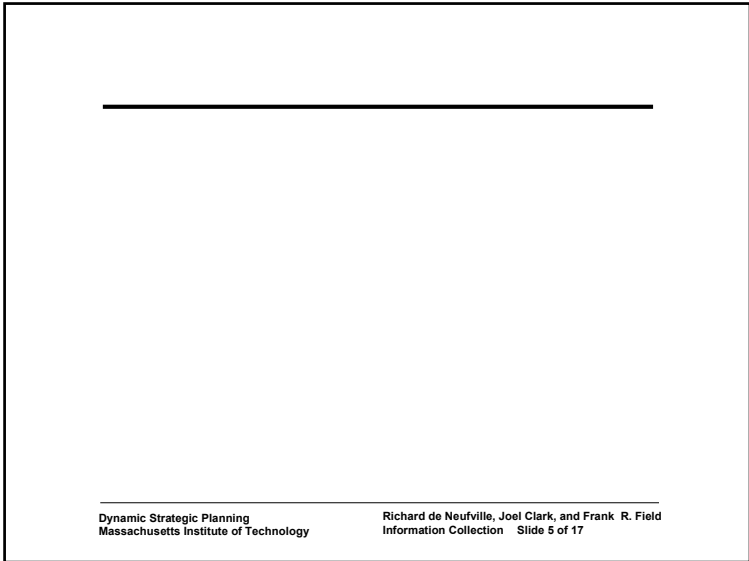
What is the value of information?  
Does it exceed the cost of the test?

## Value of Information - Essential Concept

- Value of information is an expected value
- Expected value after test “k”
  - =  $\sum_k p_k(D_k^*)$       Test  $\begin{cases} \text{Good - Revise probability} \\ \text{Medium} \\ \text{Poor} \end{cases}$
  - $p_k$  = probability, after test k, of an observation which will lead to an optimal decision (incorporating revised probabilities due to observation)  $D_k^*$
- Expected Value of information
  - =  $EV(\text{after test}) - EV(\text{without test})$
  - =  $\sum_k p_k(D_k^*) - \sum_k p_k(E_j)O_{ij}$

## Expected Value of Perfect Information - EVPI

- Perfect information is a hypothetical concept
- **Use:** Establishes an upper bound on value of any test
- **Concept:** Imagine a “perfect” test which indicated exactly which Event,  $E_j$ , will occur
  - By definition, this is the “best” possible information
  - Therefore, the “best” possible decisions can be made
  - Therefore, the EV gain over the “no test” EV must be the maximum possible - an upper limit on the value of any test!



### EVPI Example

- Question: Should I wear a raincoat?  
RC - Raincoat;  $\overline{RC}$  - No Raincoat
- Two possible Uncertain Outcomes  
( $p = 0.4$ ) or No Rain ( $p = 0.6$ )

- Remember that better choice is to take raincoat,  $EV = 0.8$

Dynamic Strategic Planning  
Massachusetts Institute of Technology

Richard de Neufville, Joel Clark, and Frank R. Field  
Information Collection Slide 6 of 17

### EVPI Example (continued)

- Perfect test

- EVPI

$$EV(\text{after test}) = 0.4(5) + 0.6(4) = 4.4$$

$$EVPI = 4.4 - 0.8 = 3.6$$

Dynamic Strategic Planning  
Massachusetts Institute of Technology

Richard de Neufville, Joel Clark, and Frank R. Field  
Information Collection Slide 7 of 17

### Application of EVPI

- A major advantage: EVPI is simple to calculate
- Notice:
  - Prior probability of the occurrence of the uncertain event must be equal to the probability of observing the associated perfect test result
  - As a "perfect test", the posterior probabilities of the uncertain events are either 1 or 0
  - Optimal choice generally obvious, once we "know" what will happen
- Therefore, EVPI can generally be written directly
- No need to use Bayes' Theorem

Dynamic Strategic Planning  
Massachusetts Institute of Technology

Richard de Neufville, Joel Clark, and Frank R. Field  
Information Collection Slide 8 of 17

## Expected Value of Sample Information - EVSI

- Sample information are results taken from an actual test  $0 \leq EVSI \leq EVPI$
- Calculations required
  - Obtain probabilities of test results,  $p_k$
  - Revise prior probabilities  $p_j$  for each test result  $TR_k \Rightarrow p_{jk}$
  - Calculate best decision  $D_k^*$  for each test result  $TR_k$  (a k-fold repetition of the original decision problem)
  - Calculate EV (after test) =  $\sum_K p_k(D_k^*)$
  - Calculate EVSI as the difference between EV (after test) - EV (without test)
- A BIG JOB

## EVSI Example

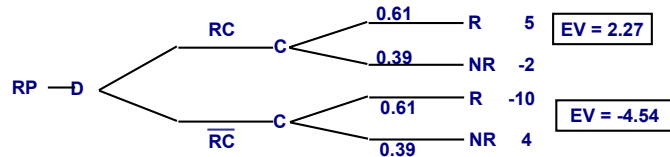
- Test consists of listening to forecasts
- Two possible test results
  - Rain predicted = RP
  - Rain not predicted = NRP
- Assume the probability of a correct forecast = 0.7
  - $p(RP/R) = P(NRP/NR) = 0.7$
  - $P(NRP/R) = P(RP/NR) = 0.3$
- First calculation: probabilities of test results
  - $P(RP) = p(RP/R) p(R) + P(RP/NR) p(NR)$   
 $= (0.7) (0.4) + (0.3) (0.6) = 0.46$
  - $P(NRP) = 1.00 - 0.46 = 0.54$

## EVSI Example (continued 2 of 5)

- Next: Posterior Probabilities
  - $P(R/RP) = p(R) (p(RP/R)/p(RP)) = 0.4(0.7/0.46) = 0.61$
  - $P(NR/NRP) = 0.6(0.7/0.54) = 0.78$
  - Therefore,  $p(NR/RP) = 0.39$  &  $p(R/NRP) = 0.22$

### EVSI Example (continued 3 of 5)

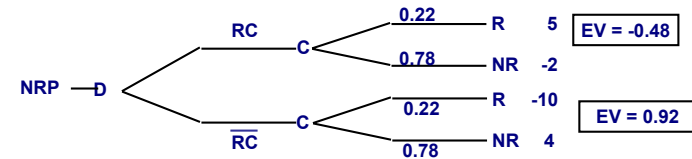
- Best decisions conditional upon test results



$$\begin{aligned} \text{EV (RC)} &= (0.61) (5) + (0.39) (-2) = 2.27 \\ \text{EV (RC)} &= (0.61) (-10) + (0.39) (4) = -4.54 \end{aligned}$$

### EVSI Example (continued 4 of 5)

- Best decisions conditional upon test results



$$\begin{aligned} \text{EV (RC)} &= (0.22) (5) + (0.78) (-2) = -0.48 \\ \text{EV (RC)} &= (0.22) (-10) + (0.78) (4) = 0.92 \end{aligned}$$

### EVSI Example (continued 5 of 5)

- EV (after test)
  - =  $p(\text{rain pred}) (\text{EV}(\text{strategy}/\text{RP}))$
  - +  $P(\text{no rain pred}) (\text{EV}(\text{strategy}/\text{NRP}))$
  - =  $0.46 (2.27) + 0.54 (0.92) = 1.54$
- EVSI =  $1.54 - 0.8 = 0.74 < \text{EVPI}$

### Practical Example - Is a Test Worthwhile?

- If value is Linear (i.e., probabilistic expectations correctly represent valuation of outcomes under uncertainty)
  - Calculate EVPI
  - If  $\text{EVPI} < \text{cost of test}$  → Reject test
  - Pragmatic rule of thumb
  - If  $\text{cost} > 50\% \text{ EVPI}$  → Reject test
  - (Real test are not close to perfect)
  - Calculate EVSI
  - $\text{EVSI} < \text{cost of test}$  → Reject test
  - Otherwise, accept test

### **Is Test Worthwhile? (continued)**

---

- **If Value Non-Linear (i.e., probabilistic expectation of value of outcomes does NOT reflect attitudes about uncertainty)**
- **Theoretically, cost of test should be deducted from EACH outcome that follows a test**
  - If cost of test is known
    - A) Deduct costs
    - B) Calculate EVPI and EVSI (cost deducted)
    - C) Proceed as for linear EXCEPT  
Question is if  $EVPI(cd)$  or  $EVSI(cd) > 0$ ?
  - If cost of test is not known
    - A) Iterative, approximate pragmatic approach must be used
    - B) Focus first on EVPI
    - C) Use this to estimate maximum cost of a test