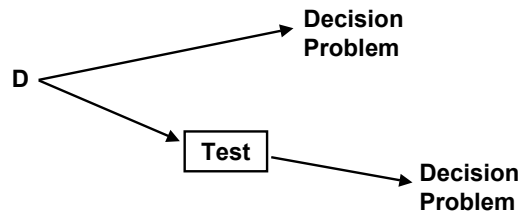
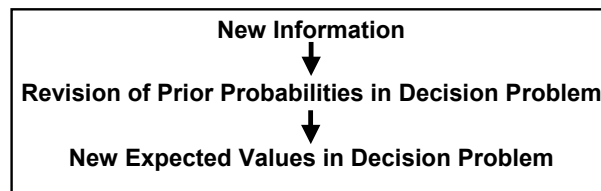


Information Collection - Key Strategy

- **Motivation**
 - To reduce uncertainty which makes us choose “second best” solutions as insurance
- **Concept**
 - Insert an information-gathering stage (e.g., a test) before decision problems, as an option



Operation of Test



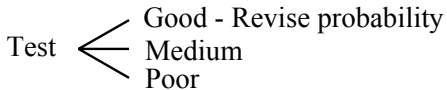
EV (after test) \geq EV (without test)

- **Why?**
 - Because we can avoid bad choices and take advantage of good ones, in light of test results
- **Question:**
 - Since test generally has a cost, is the test worthwhile?
**What is the value of information?
Does it exceed the cost of the test?**

Essential Concept

- Value of information is an expected value
- Expected value after test “k”

$$= \sum_k p_k(D_k^*)$$

Test 

P_k = probability, after test k, of an observation which will lead to an optimal decision (incorporating revised probabilities due to observation) D_k^*

- Expected Value of information

$$= \text{EV (after test)} - \text{EV (without test)}$$

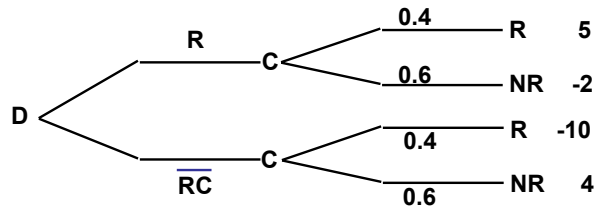
$$= \sum_k p_k(D_k^*) - \sum_k p_k(E_j)O_{ij}$$

Expected Value of Perfect Information EVPI

- Perfect information is a hypothetical concept
- Use: Establishes an upper bound on value of any test
- Concept: Imagine a “perfect” test which indicated exactly which Event, E_j , will occur
 - By definition, this is the “best” possible information
 - Therefore, the “best” possible decisions can be made
 - Therefore, the EV gain over the “no test” EV must be the maximum possible - an upper limit on the value of any test!

EVPI Example (1)

- Question: Should I wear a raincoat?
RC - Raincoat; $\bar{R}C$ - No Raincoat
- Two possible Uncertain Outcomes
($p = 0.4$) or No Rain ($p = 0.6$)



- Remember that better choice is to take raincoat, $EV = 0.8$

EVPI Example (2)

- Perfect test
-

- EVPI

$$EV(\text{after test}) = 0.4(5) + 0.6(4) = 4.4$$

$$EVPI = 4.4 - 0.8 = 3.6$$

Application of EVPI

- **A major advantage: EVPI is simple to calculate**
- **Notice:**
 - Prior probability of the occurrence of the uncertain event must be equal to the probability of observing the associated perfect test result
 - As a “perfect test”, the posterior probabilities of the uncertain events are either 1 or 0
 - Optimal choice generally obvious, once we “know” what will happen
- **Therefore, EVPI can generally be written directly**
- **No need to use Bayes’ Theorem**

Expected Value of Sample Information EVSI

- **Sample information are results taken from an actual test** $0 \leq \text{EVSI} \leq \text{EVPI}$
- **Calculations required**
 - Obtain probabilities of test results, p_k
 - Revise prior probabilities p_j for each test result $\text{TR}_k \Rightarrow p_{jk}$
 - Calculate best decision D_k^* for each test result TR_k (a k -fold repetition of the original decision problem)
 - Calculate EV (after test) = $\sum_k p_k(D_k^*)$
 - Calculate EVSI as the difference between EV (after test) - EV (without test)
- **A BIG JOB**

EVSI Example (1)

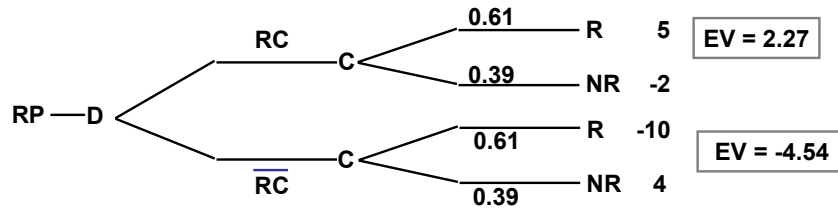
- Test consists of listening to forecasts
- Two possible test results
 - Rain predicted = RP
 - Rain not predicted = NRP
- Assume the probability of a correct forecast = 0.7
 - $p(\text{RP}/\text{R}) = P(\text{NRP}/\text{NR}) = 0.7$
 - $P(\text{NRP}/\text{R}) = P(\text{RP}/\text{NR}) = 0.3$
- First calculation: probabilities of test results
 - $P(\text{RP}) = p(\text{RP}/\text{R}) p(\text{R}) + P(\text{RP}/\text{NR}) p(\text{NR})$
 $= (0.7) (0.4) + (0.3) (0.6) = 0.46$
 - $P(\text{NRP}) = 1.00 - 0.46 = 0.54$

EVSI Example (2)

- Next: Posterior Probabilities
 - $P(\text{R}/\text{RP}) = p(\text{R}) (p(\text{RP}/\text{R})/p(\text{RP})) = 0.4(0.7/0.46) = 0.61$
 - $P(\text{NR}/\text{NRP}) = 0.6(0.7/0.54) = 0.78$
- Therefore, $p(\text{NR}/\text{RP}) = 0.39$ & $p(\text{R}/\text{NRP}) = 0.22$

EVSI Example (3)

- Best decisions conditional upon test results

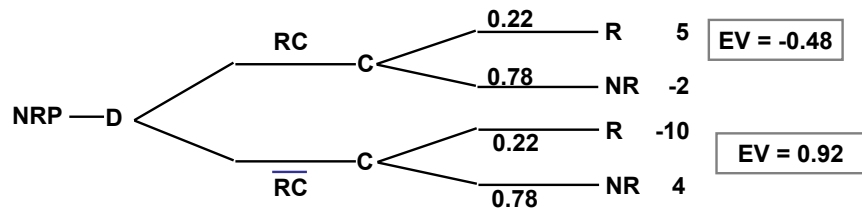


$$EV(RC) = (0.61)(5) + (0.39)(-2) = 2.27$$

$$EV(RC) = (0.61)(-10) + (0.39)(4) = -4.54$$

EVSI Example (4)

- Best decisions conditional upon test results



$$EV(RC) = (0.22)(5) + (0.78)(-2) = -0.48$$

$$EV(RC) = (0.22)(-10) + (0.78)(4) = 0.92$$

EVSI Example (5)

- EV (after test)
= $p(\text{rain pred}) (\text{EV}(\text{strategy}/\text{RP}))$
+ $P(\text{no rain pred}) (\text{EV}(\text{strategy}/\text{NRP}))$
= $0.46 (2.27) + 0.54 (0.92) = 1.54$
- $\text{EVSI} = 1.54 - 0.8 = 0.74 < \text{EVPI}$

Practical Example: Is a Test Worthwhile? (1)

- If value is Linear (i.e., probabilistic expectations correctly represent valuation of outcomes under uncertainty)
 - Calculate EVPI
 - If $\text{EVPI} < \text{cost of test}$ → Reject test
 - Pragmatic rule of thumb
 - If $\text{cost} > 50\% \text{ EVPI}$ → Reject test
(Real test are not close to perfect)
 - Calculate EVSI
 - $\text{EVSI} < \text{cost of test}$ → Reject test
 - Otherwise, accept test

Is Test Worthwhile? (2)

- **If Value Non-Linear (i.e., probabilistic expectation of value of outcomes does NOT reflect attitudes about uncertainty)**
- **Theoretically, cost of test should be deducted from EACH outcome that follows a test**
 - **If cost of test is known**
 - A) Deduct costs**
 - B) Calculate EVPI and EVSI (cost deducted)**
 - C) Proceed as for linear EXCEPT**
 - Question is if $EVPI(cd)$ or $EVSI(cd) > 0$?**
 - **If cost of test is not known**
 - A) Iterative, approximate pragmatic approach must be used**
 - B) Focus first on EVPI**
 - C) Use this to estimate maximum cost of a test**