The CAPM
(Capital Asset Pricing Model)

NPV Dependent on Discount Rate Schedule

- Discussed NPV and time value of money
- Choice of discount rate influences decisions
- WACC may be appropriate for average projects
- What discount rate applies to unique projects?

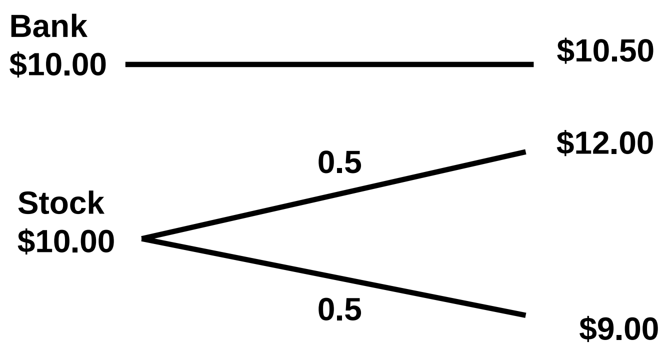
CAPM: A Basis for Adjusting Discount Rates for Risk

- Development of the Capital Asset Pricing Model
  - Assumptions about investor's view of risky investments
  - Risk characteristics and components
  - Principle of diversification
  - Beta: a formal metric of risk
  - The Capital Asset Pricing Model relationship between risk and expected return
  - The Security Market Line and expected return for individual investments
- Use of CAPM principles for project evaluation
- Comparison of utility theory and CAPM

Motivation for CAPM: Investors Prefer Less Risk

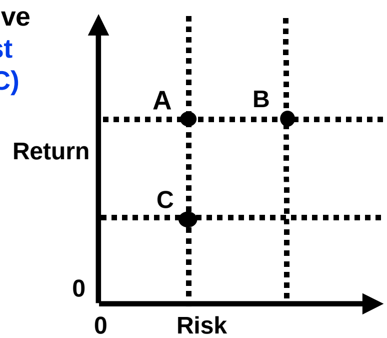
- Consider two investments
  - Deposit $10 in a savings account with annual yield of 5%
  - Buy stock for $10 with a 50 - 50 chance of selling for $12 or $9 in one year
- Which is more attractive to risk-averse investors?
  - Expected return for savings account = 5%
  - Expected return for stock = (0.5*(12+ 9) -10)/10*100%= 5%
- For same return, investors prefer less risky savings account
- What if stock had a 75% chance of selling for $12?

Motivation for CAPM (2)



How do Investors Regard Risk and Return?

- Two key observations regarding preferences
- Non-satisfaction
  - For a given level of risk, the preferred alternative is one with the highest expected return (A > C)
- Risk Aversion
  - For a given level of return, the preferred alternative is one with the lowest level of risk (A > B)



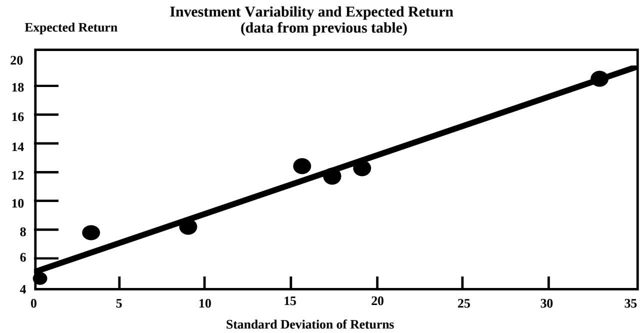
Risk Metrics: An Empirical Observation

- Investors expect compensation for variability (risk)
- Risk-free rate defined as return if no variability
- More risky securities priced to return premium
- Correlation between variability and expected return

Security	Expected Return %	Variability: Standard Deviation of Expected Returns (%)
Risk free	5	0
U.S Treasuries	7.7	3.3
Fixed Income	9.0	9.0
Domestic Equity	12.7	18.5
International Equity	12.9	19.4
Real Estate	12.9	16.9
Venture Capital	18.6	33.0

Relationship Between Variability and Expected Return

- An upward trend
- Argument based on aggregate performance of groups
- CAPM models expectations for individual investments



Components of Risk

- Finance defines 2 risks (standard deviation)
- Market Risk (systematic, non-diversifiable)
  - —Investments tend to fluctuate with outside markets
  - —Declines in the stock market and the price of Microsoft might be correlated
- Unique or Project Risk (idiosyncratic, diversifiable)
  - —Individual characteristics of investments affect return
  - —Microsoft might increase in price, despite a decline in the overall stock market
- Diversify by holding a portfolio of many investments
- What compensation should investors demand for each type?

Role of Diversification

- Unique risks are reduced by holding an investment portfolio
- Example: Two Stocks
  - —A: Expected return = 20%,
    Standard Deviation of Expected Returns = 20%
  - —B: Expected Return = 20%
    Standard Deviation of Expected Returns = 20%
- Consider portfolio with equal amounts of A and B
  - —Expected return = 0.5*20% + 0.5*20% = 20%
  - —Standard Deviation?

Standard Deviation for a Portfolio

- Portfolio standard deviation is not a weighted average
- Portfolio standard deviation

  $\sigma_p = \sqrt{\Sigma_i \Sigma_j x_i x_j \sigma_I \sigma_j \rho_{ij}}$

  for a portfolio of N investments, with i, j = 1 to N

  x_i, x_j = Value fraction of portfolio represented by investments i and j

  σ_i, σ_j = Standard deviation of investments i and j
  ρ_{ij} = Correlation between investments i and j

  ρ_{jj} = 1.0

Example: Standard Deviation for a 2 Stock Portfolio

- Invest equal amounts in two stocks
  - —For both A & B: Expected Return = 20%, Standard Deviation = 20%

$\sigma_p = \sqrt{(0.5)(0.5)(0.2)(0.2)(1)+ (0.5)(0.5)(0.2)(0.2)(1)+(2)(0.5)(0.5)(0.2)(0.2)\rho_{a1b}}$

- Portfolio standard deviation depends on correlation of A, B

Correlation Between A & B	Portfolio Standard Deviation	Portfolio Expected Return
1	20.0%	20%
0.5	17.3%	20%
0	14.1	20%
-1	0.0%	20%

Example: Standard Deviation for a 2 Stock Portfolio (2)

- Most investments not perfectly correlated (correlation < 1)
- Holding portfolio leads to risk reduction
- With negative correlation, can eliminate all risk

Generalization for Portfolio with Many Stocks

- General formula for standard deviation of portfolio returns

$\sigma p = \sqrt{\Sigma_i \Sigma_j x_i x_j \sigma_i \sigma_j \rho_{ij}} = \sqrt{\text{portfolio variance}}$

- For a portfolio of N stocks in equal proportions ($x_i = x_j = 1/N$)
  - — N weighted variance terms, $i = j \rightarrow \sigma^2_i$
  - — (N^2-N) weighted cov. terms, i ° $j \rightarrow \sigma_i \sigma_j \rho_{ij}$
- Var(P) = N*(1/N)^2* Average Variance + (N^2-N)* (1/N)^2*Average Covariance
- Var(P) = (1/N)*Av. Var. + [1-(1/N)*] Av. Cov.

Generalization for Portfolio with Many Stocks (2)

$\sigma_p = \sqrt{(1/N)^* \text{ Average Variance} + (1-(1/N)) \text{ Average Covariance}}$

- **For large N, $1/N \Rightarrow 0$**
  - — Average variance term associated with unique risks becomes irrelevant
  - — Average covariance term associated with market risk remains

Defining a Formal Measure of Risk

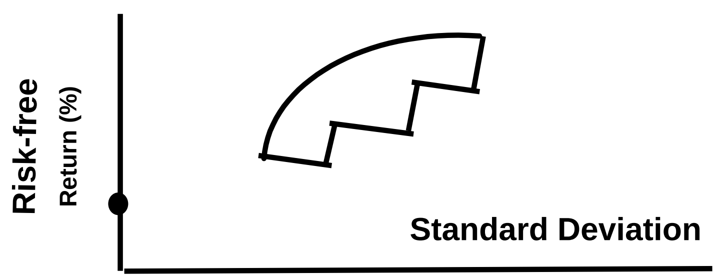
- **Investors expect compensation for systematic. undiversifiable (market) risk**
- **Standard deviation of returns reflects market & unique risks**
- **Need method to extract market portion of risk**
- **Define a reference point: the market portfolio**
  - —The full set of available securities
    - r_m = Expected return for market portfolio
    - σ_m= Standard deviation of expected returns on market portfolio
- **Beta: index of investment risk compared to market portfolio $\beta_i = \rho_{i,m}\sigma_i/\sigma_m$**

What Does Beta Imply?

- By definition, the market portfolio has beta = 1.0
- Beta describes the relative variability of returns
  - —Concerned with correlated (systematic) portion of returns
  - —If investment amplifies movements in market portfolio beta > 1
  - —If attenuates, movements in market portfolio beta < 1
- Beta reflects market risk of an investment
  - —Investors expect higher returns for increased market risk
  - —Thus, higher returns for investments with higher betas
- Can be calculated for individual investments or portfolios
- Portfolio beta = weighted average of individual betas

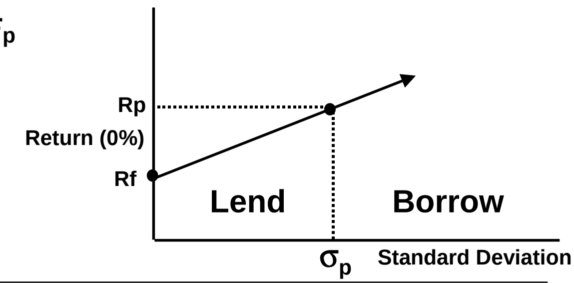
Investment Portfolios and the Efficient Frontier

- Example demonstrated role of diversification
- Combinations of many securities result in optimum
  - – Maximum return for given risk level
  - – Minimum risk for given level of return
  - – Assumes no borrowing or lending
- Sub-optimal combinations lie below, to right of frontier



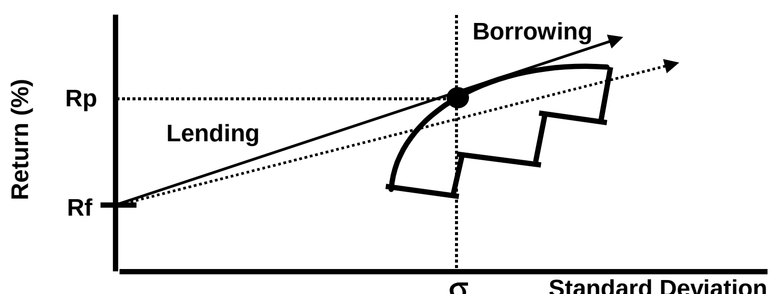
Combining Risk-Free and Risky Investments

- For any combination of risk-free and risky investing
  - —Investor can mix investments in portfolio and risk-free to achieve desired return
  - —Expected return is weighted average of risk-free (Rf) and portfolio return (Rp)
  - —Standard deviation of Rf = 0
  - —$\sigma_{mix} = x_p \sigma_p$



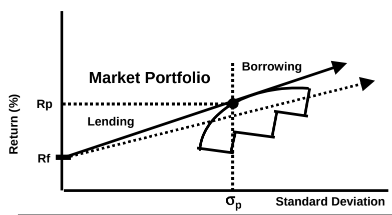
CAPM: Selecting a Portfolio to Maximize Returns for Risk

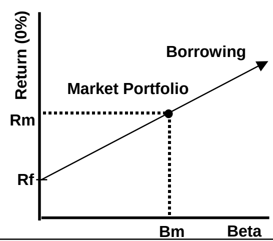
- Infinite number of portfolios, even on efficient frontier
- Tangent point yields optimum
- CAPM shows expected return for investment combinations



Determining Expected Return for Individual Investments

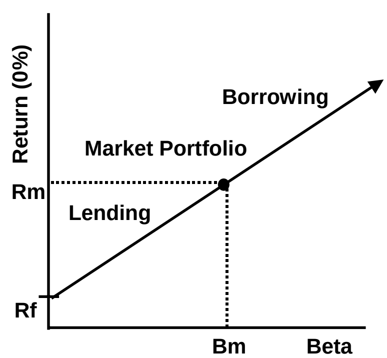
- CAPM models maximized expected return
- Beta indexes risk of individual investment to market portfolio
- Market portfolio is tangent point in CAPM
- Relation between beta and individual expected return results





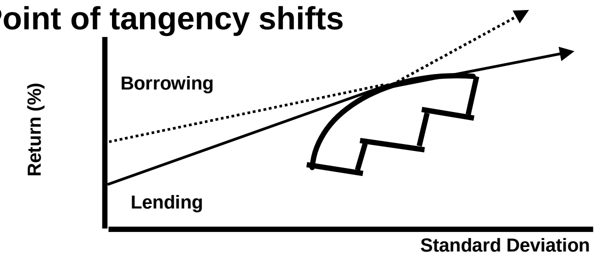
How Does Expected Return Relate to Beta?

- Security Market Line (SML)
  - —Rp = Rf + Bp*(Rm - Rf)
  - —Rm - Rf is the market risk premium
  - —Bp is the beta of the portfolio or investment to be evaluated
- For the market portfolio, Bm = Bp = 1
  - —Total expected return is Rm
- For other investments, expected return scales with Bp



Differences Between Borrowing and Lending Rates

- Not typical to have same rate for borrowing and lending
- Risk-free rate generally unattainable for small investors
- Adjustments to model possible, minor, illustrated below
- Point of tangency shifts



Implementing the CAPM: From Theory to Project Evaluation

- Relation between market risk and expected return
  - —Investments have market risk and unique risk components
  - —Market risk commands premium over risk-free rate
  - —Unique risk is managed (averaged out) by diversification
- Project discount rate should be based on project beta
  - —Investors can diversify away unique project risks
  - —Adjustment apparent if project is carbon-copy of firm (McDonald's #10,001) ==> WACC applies
- "Proper" adjustment not trivial on most projects
  - —Consider past experiences, returns in comparable industries
  - —Detail unique aspects of specific project
  - —Apply information to adjust discount rate

A General Rule for Managers

- Portfolio theory translates to a simple rule for managers:
- Use risk adjusted discount rate to calculate NPV for projects,
- Accept all positive NPV projects to maximize value
  - —Shareholders capable of diversifying unique risks by holding multiple assets
  - —Positive NPV implies market risk in projects is expected to be compensated
  - —If projects are properly valued, shareholder wealth is maximized

Limitations and Conflicts in Practice

- Estimating project beta may not trivial
- Budget constraints conflict with positive NPV rule
- Employees worry about unique project risks
  - —Career can be adversely affected by bad outcomes
  - —Not always in a position to diversify (limited to few projects)
  - —Issue might be addressed through proper incentives
- Reliance on past performance to dictate future choices
- Individuals and companies are often “risk positive”
  - —Entrepreneurs
  - —Sometimes only choice is bet the company

How does Utility Theory Compare with CAPM?

- **Utility**
  - —Applies a single discount rate for time value
  - —Adjusts for risk preference of decision-maker
  - —Utility is bottom-up and focused on individual preferences
- **CAPM**
  - —Adjusts discount rate for overall aversion to market risk
  - —No adjustment for risk preferences of decision-maker
  - —Based on top-down, aggregate perspectives
- **Utility and CAPM**
  - —Both value risky opportunities, accounting for risk aversion
  - —Under the right circumstances, should give same results
  - —"No double counting"

Summary

- **CAPM adjusts discount rates for risk**
  - —Models maximum expected return for level of risk
  - —Based on observations of securities markets
- **Unique risks can be diversified**
- **Investors expect compensation for market risk**
- **Standard deviation of returns reflects both market & unique**
- **Beta is index of market part of investment risk**
- **Security Market Line relates expected return to beta**
  - —Rp = Rf + Bp*(Rm - Rf)
- **Moving from theory to practice can be problematic**