Risk Assessment

- The quantified description of the uncertainty concerning situations and outcomes
- Objective: To present
  - The problem
  - Means of assessment
  - Useful formulas
  - Biases in assessment

Methods of Assessment (1)

- Logic
  - Example: Prob (Queen) in a deck of cards
- Frequency
  - Example:
    Prob (failure of dams) = 0.00001/dam/year
  - Based on analysis of data on failures

Methods of Assessment (2)

- Statistical Models
  - Example: Future Demand = f(variables) + error
- Judgement
  - “Expert Opinion”
  - “Subjective Probability”
  - Example:
    Performance in 10 years of a new technology
    Major War in the Middle East

Biases in Subjective Probability Assessments

- Overconfidence
  - Distribution typically much broader than we imagine
- Insensitivity to New Information
  - Information typically should cause us to change opinions more than it does

Revision of Estimates - Bayes Theorem

- Definitions
  - P(E) Prior Probability of Event E
  - P(E/O) Posterior P(E), after observation O is made. This is the goal of the analysis.
  - P(O/E) Conditional probability that O is associated with E
  - P(O) Probability of Event (Observation) O
- Theorem: P(E/O) = P(E) {P(O/E) / P(O) }
- Note: Importance of revision depends on:
  - rarity of observation O
  - extremes of P(O/E)

Application of Bayes Theorem

- At a certain educational establishment:

  P(students) = 2/3 P(staff) = 1/3

  P(fem/students) = 1/4 P(fem/staff) = 1/2

- What is the probability that a woman on campus is a student?
  {i.e., what is P(student/fem)?}

$$P(student/fem) = P(student) \frac{P(fem/student)}{P(fem)}$$

- Thus: P(student/fem) = 2/3 {(1/4) / 1/3)} = 1/2

Likelihood Ratios, LR

Definitions

$P(\bar{E})$	= P(E does not occur)
	=> $P(E) + P(\bar{E}) = 1.0$
LR	= $P(E)/P(\bar{E})$; therefore
P(E)	= LR / (1 + LR)
LR_i	= LR after i observations

Likelihood Ratios (2)

- **Formulas**

$$LR_1 = \frac{P(E)\,\{P(Oj/E) / P(Oj)\}}{P(\bar{E})\,\{P(Oj/\bar{E}) / P(Oj)\}}$$

after a single observation Oj

CLR_i = $P(Oj/E) / P(Oj/\bar{E})$

the conditional likelihood ratio for Oj

$$LR_N = LR_0 \prod_j (CLR_j)^{N_j}$$

N_j = number of observations of type O_j

Application of Likelihood Ratios (1)

- Bottle-making machines can be either OK or defective   P(D) = 0.1
- The frequency of cracked bottles depends upon the state of the machine

$$P(C/D) = 0.2$$
$$P(C/OK) = 0.05$$

Application of Likelihood Ratios (2)

Picking up 5 bottles at random from a machine, we find {2 cracked, 3 uncracked}
What is the Prob(machine defective)

$$LR_O = P(D) / P(OK) = 0.1/0.9 = 1/9$$
$$CLR_C = 0.2/0.05 = 4$$
$$CLR_{uc} = 0.8/0.95 = 16/19$$
$$LR_5 = (1/9)\,(4)^2\,(16/19)^3 = 1.06$$
$$P(D/\{2C, 3UC\}) = 0.52 = 1.06/(1 + 1.06)$$