Marginal Analysis

Outline

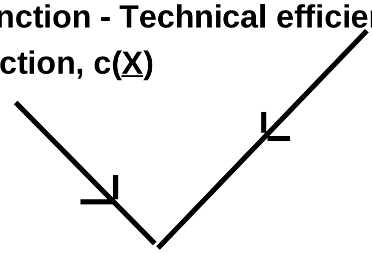
1. Definition
2. Assumptions
3. Optimality criteria
   - Analysis
   - Interpretation
   - Application
4. Expansion path
5. Cost function
6. Economies of scale

Marginal Analysis

- **Basic form of optimization of design**
- **Combines:**

  Production function - Technical efficiency

  Input cost function, c(<u>X</u>)



Economic efficiency

Assumptions of Marginal Analysis

- **Feasible region is convex (over relevant portion)**
- **No constraints on resources**
- **Models are analytic (needed only for derivation)**

Optimality Conditions for Design, by Marginal Analysis

The Problem:

Min $C(Y') = c(\underline{X})$ cost function

s.t. $g(\underline{X}) = Y'$ production function

vector of resources ($\underline{X}$); output (Y')

The Lagrangean:

$$L = c(\underline{X}) - \lambda [g(\underline{X}) - Y']$$

Optimality Conditions for Design, by Marginal Analysis (cont'd)

Key Result:

$$\partial c(\underline{X}) / \partial X_i = \lambda \, \partial g(\underline{X}) / \partial X_i$$

marginal cost — marginal product

Optimality Conditions:

$MP_i / MC_i = 1 / \lambda = MP_j / MC_j$

or $MP_i / MP_j = MC_i / MC_j$

A **balanced design**

Each X_i contributes

"same bang for buck"

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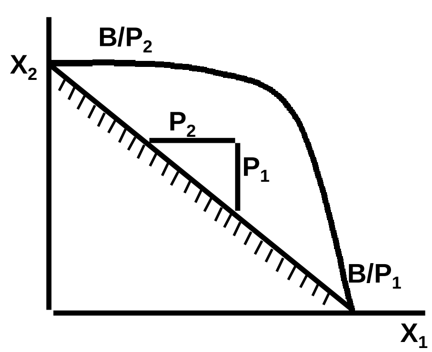
Graphical Interpretation of Optimality Conditions

(A) Input Cost Function



B = Budget
$c(\underline{X}) = \Sigma p_i X_i \leq B$

Linear case:
In general, non-linear
(as in curved line)

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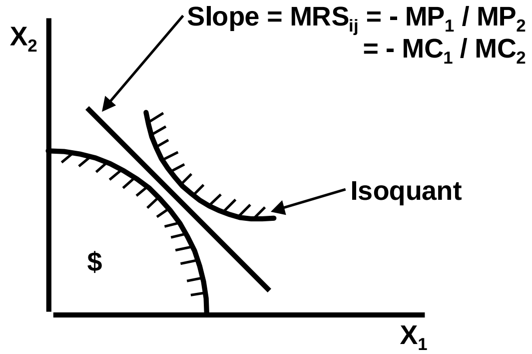
Graphical Interpretation of Optimality Conditions (cont'd)

(B) Conditions



Application of Optimality Conditions

Problem: $Y = a_0 X_1^{\,a1} X_2^{\,a2}$

$c(\underline{X}) = \Sigma p_i X_i$

Note: Linearity of Input Cost Function
- typically assumed by economists
- in general, not valid
  - prices rise with demand
  - wholesale, volume discounts

Solution:

$$[a_1 / X_1^*]\ Y / p_1 = [a_2 / X_2^*]\ Y / p_2$$

(* denotes an optimum value)

Expansion Path

- Locus of all optimal designs $\underline{X}$*
- Not a property of technical system alone
- Depends on local prices
- Optimal designs do not, in general, maintain constant ratios between optimal X_i*

  e.g.: crew of 20,000 ton ship
  crew of 200,000 ton ship

Calculation of Expansion Path

Assume: $Y = 2X_1^{0.48}X_2^{0.72}$

$c(\underline{X}) = X_1 + X_2^{1.5}$

(increasing RTS)

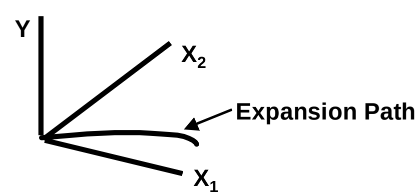
Optimality Conditions:

$(0.48 / X_1)\ Y / 1 = (0.72 / X_2)\ Y / (1.5X_2^{0.5})$

$= MP_i / MC_i$

$\Rightarrow X_1^* = (X_2^*)^{1.5}$

Graphically:



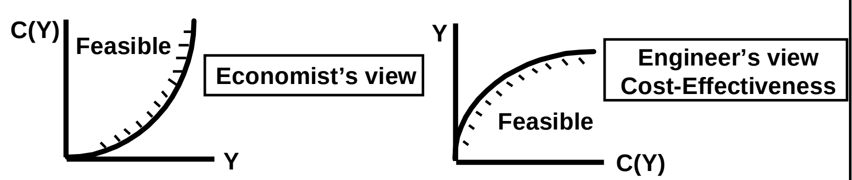
Cost Functions

- $C(Y) = C(\underline{X}^*) = f(Y)$

- **Not same as input cost function**
  - **optimal cost of Y**
  - **vs. cost of any X**

Cost Functions (cont'd)

- **Graphically:**



- **Great practical use:**
  - **How much Y for budget?**
  - **ΔY for ΔB?**
  - **Cost effectiveness, ΔB / ΔY**

Calculation of Cost Function

- **Cobb-Douglas production function**

  $Y = a_0 \pi X_i^{a_i}$

- **Linear input cost function**

  $c(X) = \Sigma p_i X_i$

- **Result**

  $C(Y) = A(\pi p_i^{a_i/r}) Y^{1/r}$

  where $r = \Sigma a_i$

- **Easy to estimate statistically**

  => Solution for 'a_i'

  => Estimate of production function

  $Y = a_0 \pi X_i^{a_i}$

Calculation of Cost Function (cont'd)

- **Assume Again:**

  $Y = 2X_1^{0.48} X_2^{0.72}$

  $c(\underline{X}) = X_1 + X_2^{1.5}$

- **Expansion Path:** $X_1^* = (X_2^*)^{1.5}$

  Thus: $Y = 2(X_2^*)^{1.44}$

  $c(\underline{X}^*) = 2(X_2^*)^{1.5}$

  => $X_2^* = (Y/2)^{0.7}$

  $c(Y) = c(\underline{X}^*) = (2^{-0.05})Y^{1.05}$

Economies of Scale

- **A possible characteristic of cost function**
- **Concept similar to returns to scale, except**
  - **ratio of 'X_i' not constant**
  - **refers to costs (economies)**
- **Economies of scale exist if costs increase slower than product**

  Total cost = $C(Y) = Y^{\propto}$ $\propto < 1.0$

Economies of Scale (cont'd)

- **Note:**

  If Cobb-Douglas, linear input costs

  Increasing RTS

  => Economies of scale

  $r = \Sigma a_i > 1.0$ => $C(Y) = \text{fcn } Y^{1/r}$

Not necessarily true in general

See other example!!