SMA ADVANCED MATERIALS PROGRAM
MATERIALS SELECTION, DESIGN AND ECONOMICS
SMA 5103 & MIT 3.57
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MATERIALS SELECTION IN MECHANICAL DESIGN

PART 3

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1. A material has certain property attributes – its density, stiffness, strength, cost, corrosion resistance, etc.

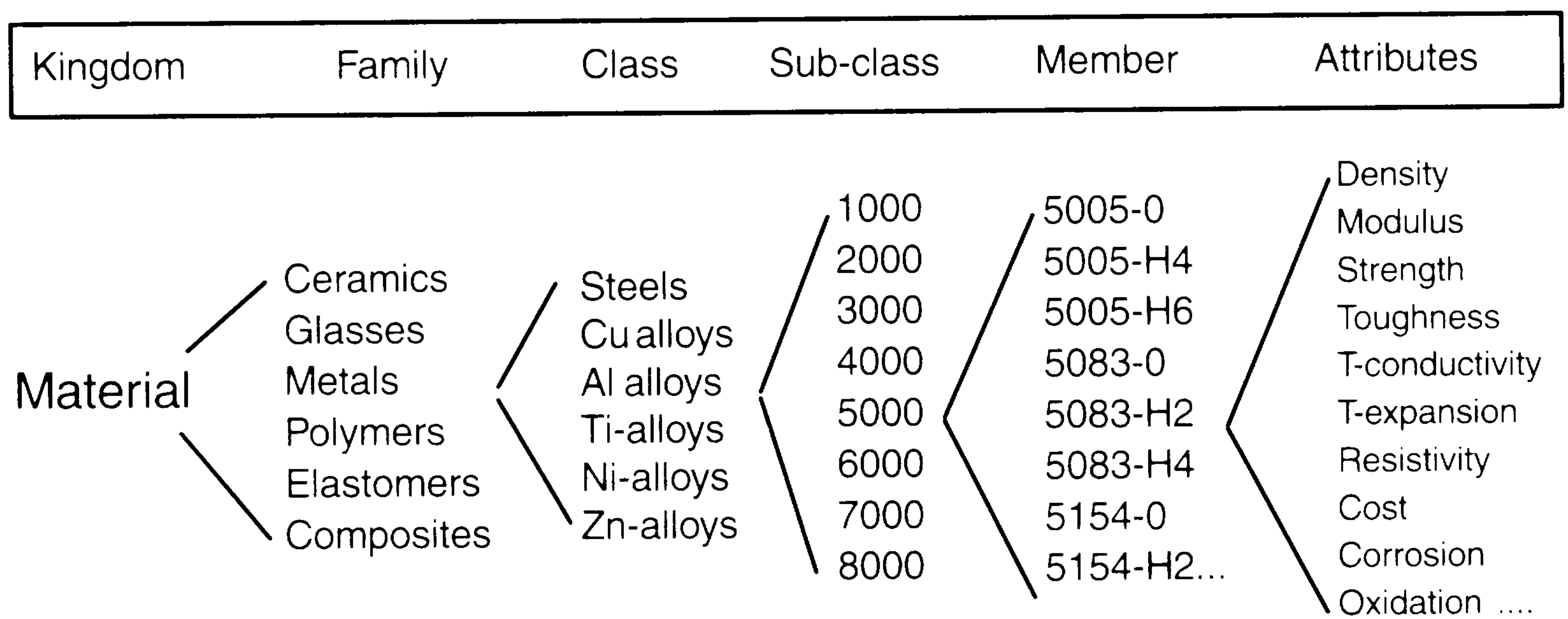
2. A design demands certain profile of these: a low density, a high strength, a modest cost, resistance to sea water, etc.

3. The problem is to identify the desired attribute profile, and then compare it with those of real engineering materials to find the best match. This is done by

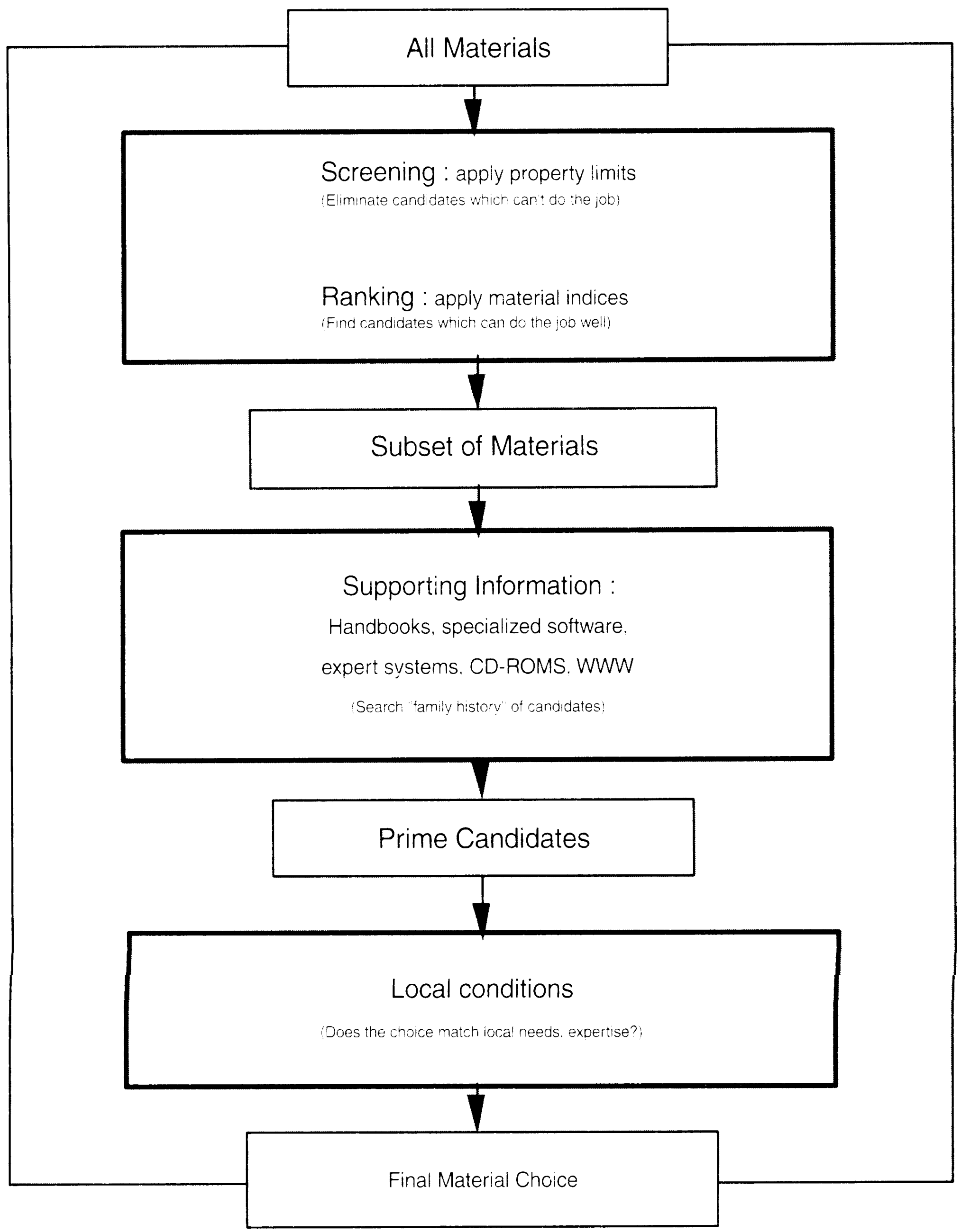
   (a) **Screening** by applying **property limits**.

   (b) **Ranking** the candidates by ability to maximize performance, by applying **material indices**.

   (c) Seeking **supporting information** for each shortlisted candidate, by consulting handbooks, specialized software, www, etc.



The taxonomy of the kingdom of materials and their attributes.



The strategy for materials selection. The main steps are enclosed in bold boxes.

Property Limits and Material Indices

These are determined by looking at

1. The **function** of the component. For example, to support a load, contain a pressure, transmit heat, etc.

2. The **objectives** that the designer has selected to optimize its performance. For example, to make it as cheap or possible, or as lightweight as possible, or safe as possible, or some combinantion of these three.

3. The **constraints** that the component must meet. For example, certain dimensions are fixed, it has to function in a certain temperature range, etc.

- **Function**:

  What does a component do?

- **Objective**:

  What is to be minimized or maximized?

- **Constraints**:

  What conditions must absoloutely be met? What conditions are desirable, but negotiable?

Property Limits

Some constraints translate directly into **limits on material properties**.

For example, if a component must operate at 250°C, then all components with a maximum service temperature less than this are eliminated.

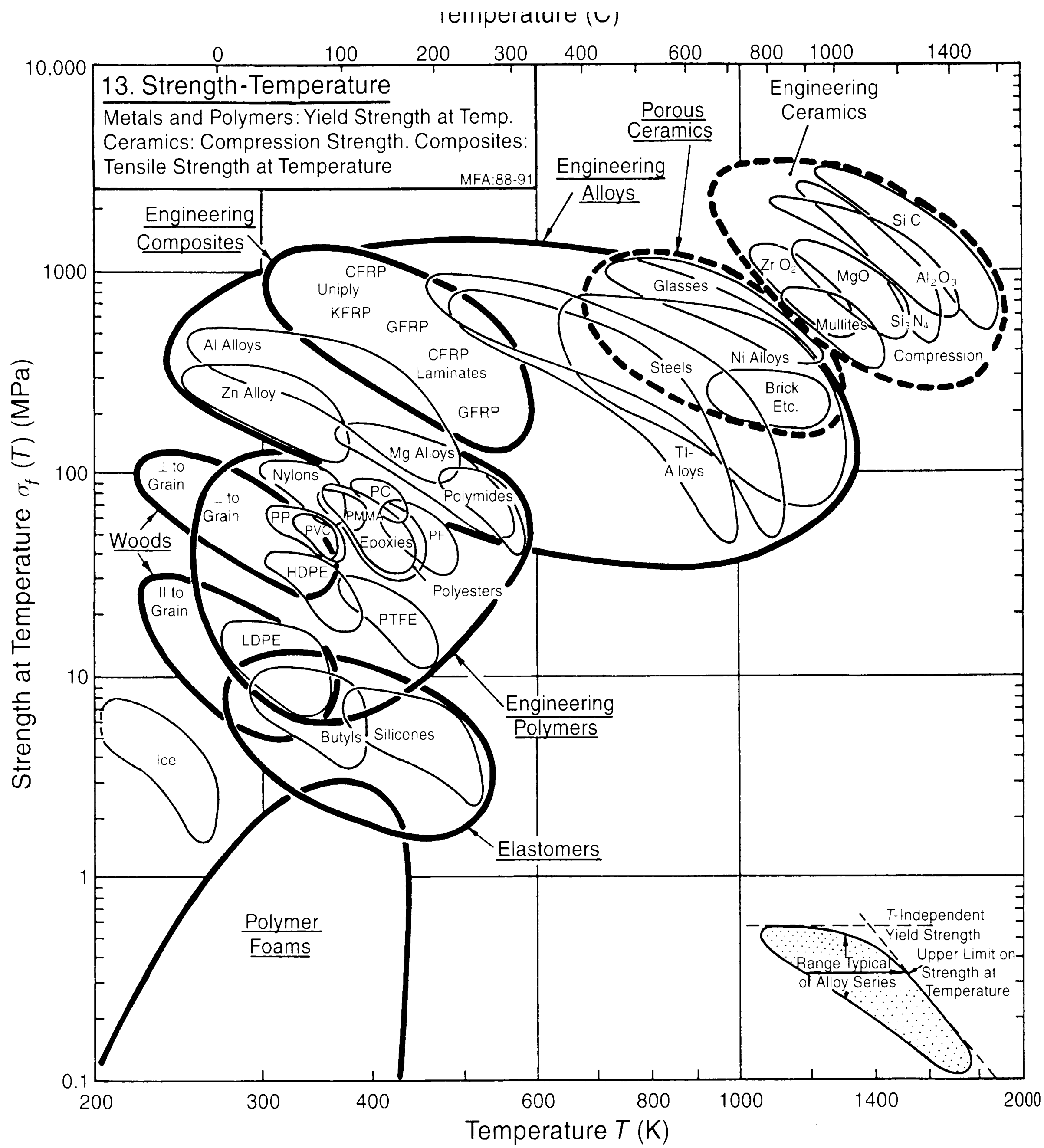


Chart 13: Strength plotted against temperature. The inset explains the shape of the lozenges.

Material Indices

Definition: A **material index** is a combination of material properties which characterizes the performance of a given material in a given application.

$$
\begin{aligned}
\text{Performance} = f\,[&(\text{Functional requirements}, F)\,, \\
&(\text{Geometric parameters}, G)\,, \\
&(\text{Material Properties}, M)] \\
p = f(&F, G, M)
\end{aligned}
$$

An **optimum design** is the selection of the material and the geometry which maximizes p.

$$p = f(F, G, M)$$

Under certain circumstances the function f is **separable**:

$$p = f_1(F) f_2(G) f_3(M).$$

Under these conditions,

$$\textbf{Structural Index} \equiv f_1(F) f_2(G),$$

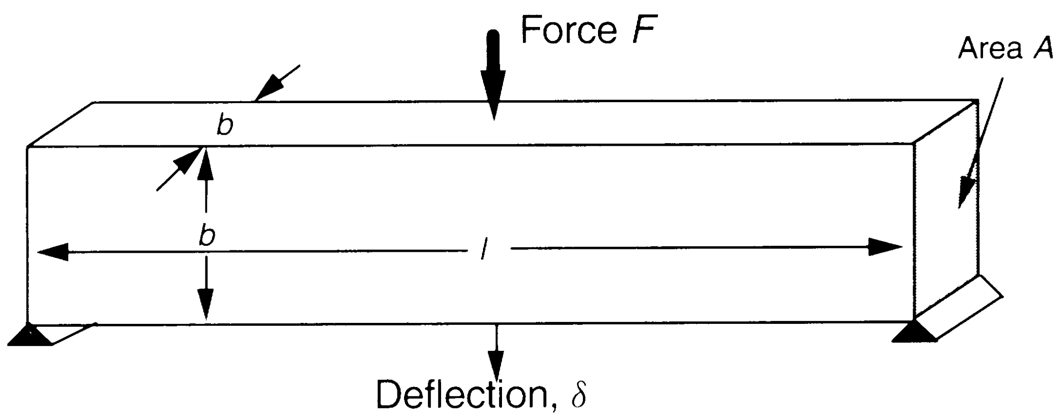
and

$$\textbf{Material Index} \equiv f_3(M).$$

In this case the optimum choice of material becomes independent of the details of the design: the performance for all values of F and G can be maximized by maximizing the **material index**, $f_3(M)$!

EXAMPLE

Select a material for a light stiff beam of length l, and a square cross-section of edge-length b, to support a bending load F without deflecting too much. That is, the stiffness S is specified. It is to be of minimum mass.



Design requirements for a light stiff beam:

- **Function**: Beam

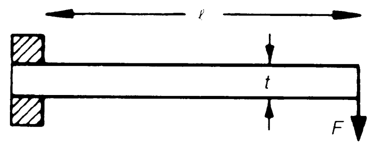
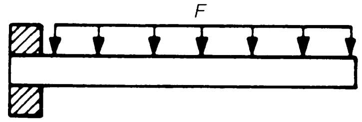
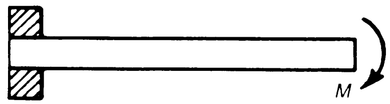
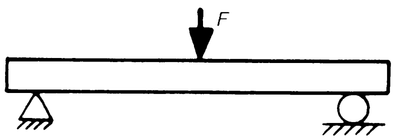
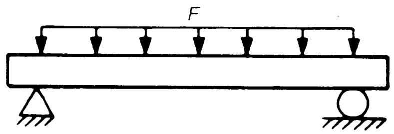
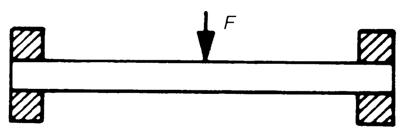
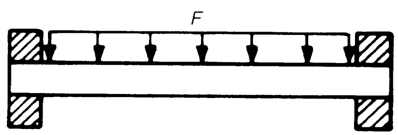
- **Objective**: Minimize the mass

- **Constraints**:

  1. Length l specified. Square cross-section of edge-length b.

  2. Stiffness S specified

Elastic bending of beams

	C_1	C_2
	3	2
	8	6
	2	1
	48	16
	$\frac{384}{5}$	24
	192	–
	384	–
	6	–
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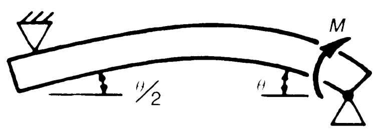
	–	3

$$\delta = \frac{F\ell^3}{C_1 EI} = \frac{M\ell^2}{C_1 EI}$$

$$\theta = \frac{F\ell^2}{C_2 EI} = \frac{M\ell}{C_2 EI}$$

E = Young's modulus (N/m^2)

δ = deflection (m)

F = force (N)

M = moment (Nm)

ℓ = length (m)

b = width (m)

t = depth (m)

θ = end slope (–)

I = see Table 2 (m^4)

y = distance from N.A. (m)

R = radius of curvature (m)

$$\frac{\sigma}{y} = \frac{M}{I} = \frac{E}{R}$$

Minimize

$$m = A\,l\,\rho \quad \text{Objective function,}$$

subject to

$$\frac{F}{\delta} = \frac{C_1 E I}{l^3} \geq S \quad \text{Constraint,}$$

where A is the cross-sectional area of the beam, and where

$$I = \frac{b^4}{12} = \frac{A^2}{12}$$

is the area moment of inertia.

$$\frac{C_1 EI}{l^3} \geq S \quad \Rightarrow \quad \frac{C_1 E}{l^3}\frac{A^2}{12} \geq S \quad \Rightarrow \quad A \geq (12S)^{(1/2)} \left(\frac{l^3}{C_1}\right)^{1/2} \frac{1}{E^{1/2}}$$

The smallest cross-sectional area A which meets the stiffness constraint is

$$A = (12S)^{(1/2)} \left(\frac{l^3}{C_1}\right)^{1/2} \frac{1}{E^{1/2}}$$

Substituting this in the objective function $m = Al\rho$ gives

$$m = \left\{(12S)^{1/2}\right\} \left\{\left(\frac{l^5}{C_1}\right)^{1/2}\right\} \left\{\left(\frac{\rho}{E^{1/2}}\right)\right\}$$

$$m = \{\text{Functional Req.}\} \{\text{Specified Geom.}\} \{\text{Material Properties}\}$$

Thus the mass will be minimized by selecting materials with the largest value of the index

$$\boxed{M = \frac{E^{1/2}}{\rho}} \qquad \text{Material Index}$$

In deriving this index, we have assumed that the section of the beam remained square so that both edges changed in length when A changed. However, if one of the two edges is held fixed, the the index changes. For example, consider the case, where only the height is free.

Design requirements for a light stiff beam:

- **Function**: Beam

- **Objective**: Minimize the mass

- **Constraints**:

  1. Length l specified. Depth b is specified.

  2. Stiffness S specified

Minimize

$$m = b\,h\,l\,\rho \quad \text{Objective function,}$$

subject to

$$\frac{C_1 E I}{l^3} \geq S \quad \text{Constraint,}$$

where

$$I = \frac{bh^3}{12}$$

is the area moment of inertia.

$$\frac{C_1 EI}{l^3} \geq S \quad \Rightarrow \quad \frac{C_1 E}{l^3}\frac{bh^3}{12} \geq S$$

The smallest h which meets the stiffness constraint is

$$h = (12S)^{(1/3)} \left(\frac{1}{C_1 b}\right)^{1/3} l \frac{1}{E^{1/3}}$$

Substituting this in the objective function $m = bhl\rho$ gives

$$m = \left\{(12S)^{1/3}\right\} \left\{\left(\frac{b^2}{C_1}\right)^{1/3} l^2\right\} \left\{\left(\frac{\rho}{E^{1/3}}\right)\right\}$$

$$m = \{\text{Functional Req.}\}\,\{\text{Specified Geom.}\}\,\{\text{Material Properties}\}$$

Thus when the height h is free and the width b fixed, the mass will be minimized by selecting materials with the largest value of the index

$$\boxed{M = \frac{E^{1/3}}{\rho}}$$

If the width b is free and h is fixed, it becomes

$$\boxed{M = \frac{E}{\rho}.}$$

EXAMPLE

Select a material for a light strong beam of length l, and a square cross-section of edge-length b, to support a bending load F without failing due yield. It is to be of minimum mass.

Design requirements for a light strong beam:

- **Function**: Beam
- **Objective**: Minimize the mass
- **Constraints**:
  1. Length l specified. Square cross-section of edge-length b.
  2. Support load F without yield.

Minimize

$$m = b^2 \, l \, \rho \quad \text{Objective function,}$$

subject to

$$|\sigma_{max}| \leq \sigma_y \quad \text{Constraint,}$$

where

$$|\sigma_{max}| = \frac{M_{max} \, y_{max}}{I} = \frac{(Fl/2)(b/2)}{(b^4/12)} = \frac{3Fl}{b^3}.$$

Hence, the constraint requires

$$\frac{3Fl}{b^3} \leq \sigma_y, \quad \Rightarrow \quad b \geq (3F)^{1/3} \, l^{1/3} \frac{1}{\sigma_y^{1/3}}$$

The smallest depth b which meets the strength constraint is

$$b = (3F)^{1/3} \, l^{1/3} \frac{1}{\sigma_y^{1/3}}$$

Substituting this in the objective function $m = b^2 l \rho$ gives

$$m = \left\{(3F)^{2/3}\right\}\left\{l^{5/3}\right\}\left\{\left(\frac{\rho}{\sigma_y^{2/3}}\right)\right\}$$

$$m = \{\text{Functional Req.}\}\{\text{Specified Geom.}\}\{\text{Material Properties}\}$$

Thus the mass will be minimized by selecting materials with the largest value of the index

$$\boxed{M = \frac{\sigma_y^{2/3}}{\rho}} \qquad \text{Material Index}$$

EXAMPLE

Select a material for a solid cylindrical rod of length L to withstand a compressive force F without buckling. It is to be of minimum cost.

Design requirements for a cheap column:

- **Function**: Column
- **Objective**: Minimize the cost
- **Constraint**:
  1. Length L specified
  2. Support compressive load F without buckling

Minimize

$$C = A\,L\,\rho\,C_m \quad \text{Objective function,}$$

where C_m is the cost per kg of the processed material from which the column is made. This is subject to

$$F \leq F_{cr} \quad \text{Constraint,}$$

where

$$F_{cr} = c\,\frac{\pi^2 E I}{L^2},$$

and

$$I = \frac{\pi R^4}{4} = \frac{A^2}{4\pi}$$

is the area moment of inertia, and c is the end-fixity coefficient (this is not important here!)

$$F \leq F_{cr} \quad \Rightarrow \quad F \leq c\,\frac{\pi^2 EI}{L^2} \quad \Rightarrow \quad F \leq c\,\frac{\pi^2 E}{L^2}\frac{A^2}{4\pi}$$

or

$$A^2 \geq 4F\left(\frac{L^2}{c\pi}\right)\frac{1}{E} \quad \Rightarrow \quad A \geq 2F^{1/2}\left(\frac{L^2}{c\pi}\right)^{1/2}\frac{1}{E^{1/2}}$$

Since L is fixed, to minimize mass choose

$$A = 2F^{1/2}\left(\frac{L^2}{c\pi}\right)^{1/2}\frac{1}{E^{1/2}}.$$

Subsitute this in the objective function

$$C = \left\{\left(\frac{2}{\pi^{1/2}}\right)F^{1/2}\right\}\left\{\left(\frac{L^4}{c}\right)^{1/2}\right\}\left\{\left(\frac{C_m\rho}{E^{1/2}}\right)\right\}$$

$m = \{\text{Functional Req.}\}\{\text{Specified Geom.}\}\{\text{Material Properties}\}$

Thus the mass will be minimized by selecting materials with the largest value of the index

$$\boxed{M = \frac{E^{1/2}}{C_m \rho}} \qquad \text{Material Index}$$

Properties like modulus, strength or conductivity do not change with time. However, cost does! To make **some** correction for the influence of inflation and currency fluctuations we may define a **relative cost** C_R:

$$C_R = \frac{\text{cost per kg of the material}}{\text{cost per kg of mild steel rod}}.$$

Steel rod at the present time (late 1999) costs about \$0.3/kg.

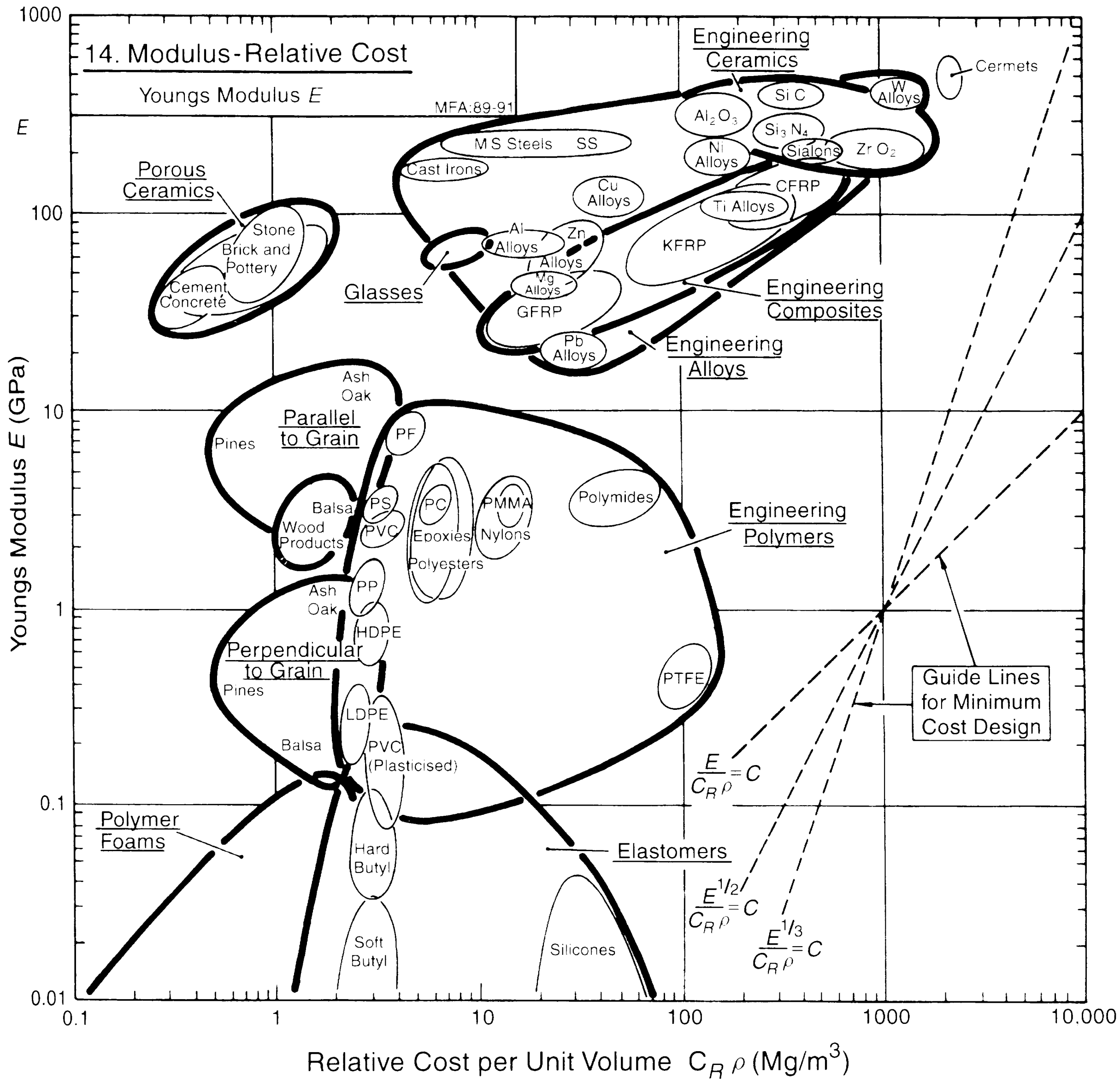


Chart 14: Young's modulus, E, plotted against relative cost per unit volume, $C_p\rho$. The desigr guide lines help selection to maximize stiffness per unit cost.

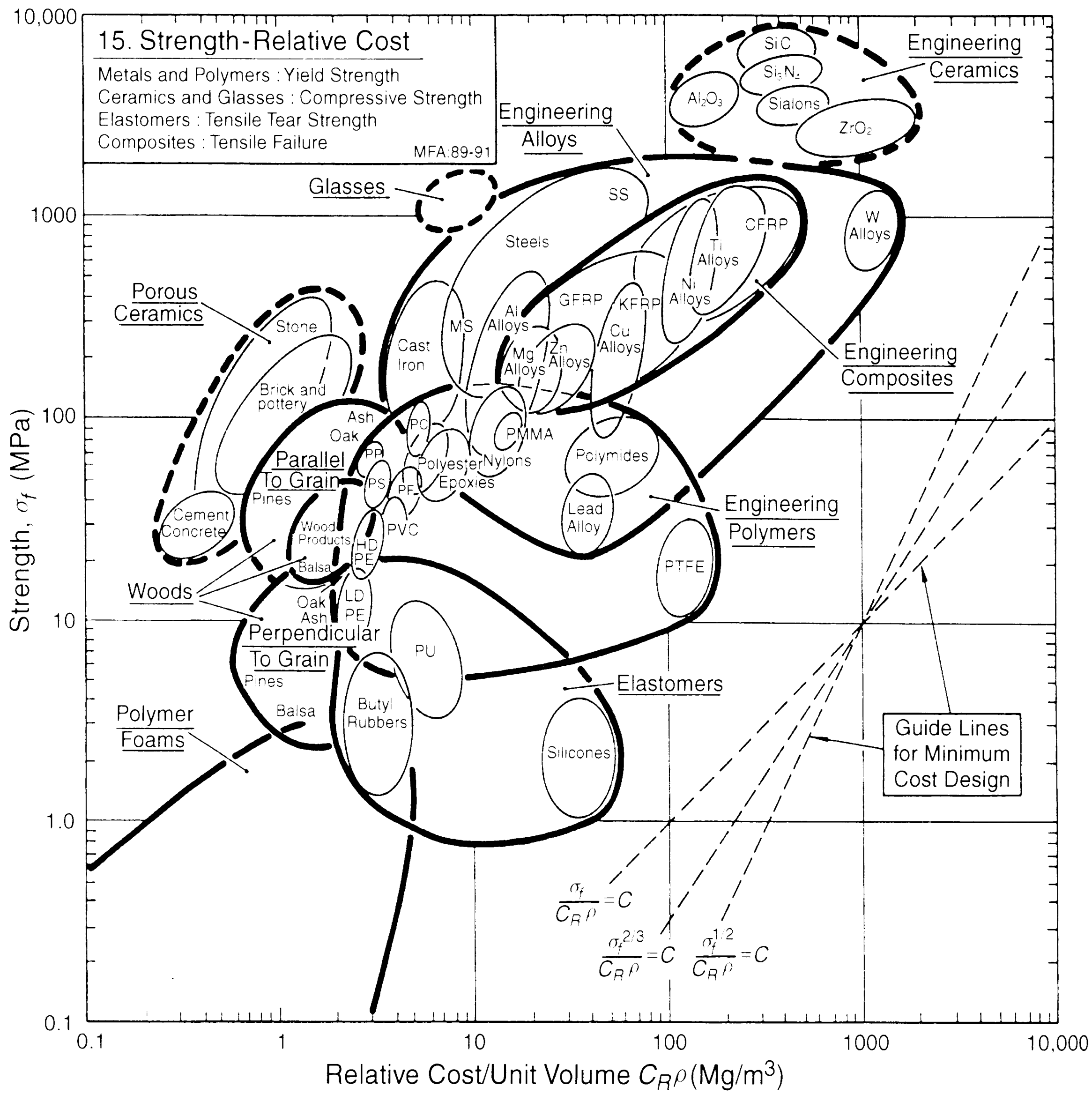


Chart 15: Strength, σ_f, plotted against relative cost per unit volume, $C_p \rho$. The design guide lines help selection to maximize strength per unit cost.

Procedure for deriving material indices

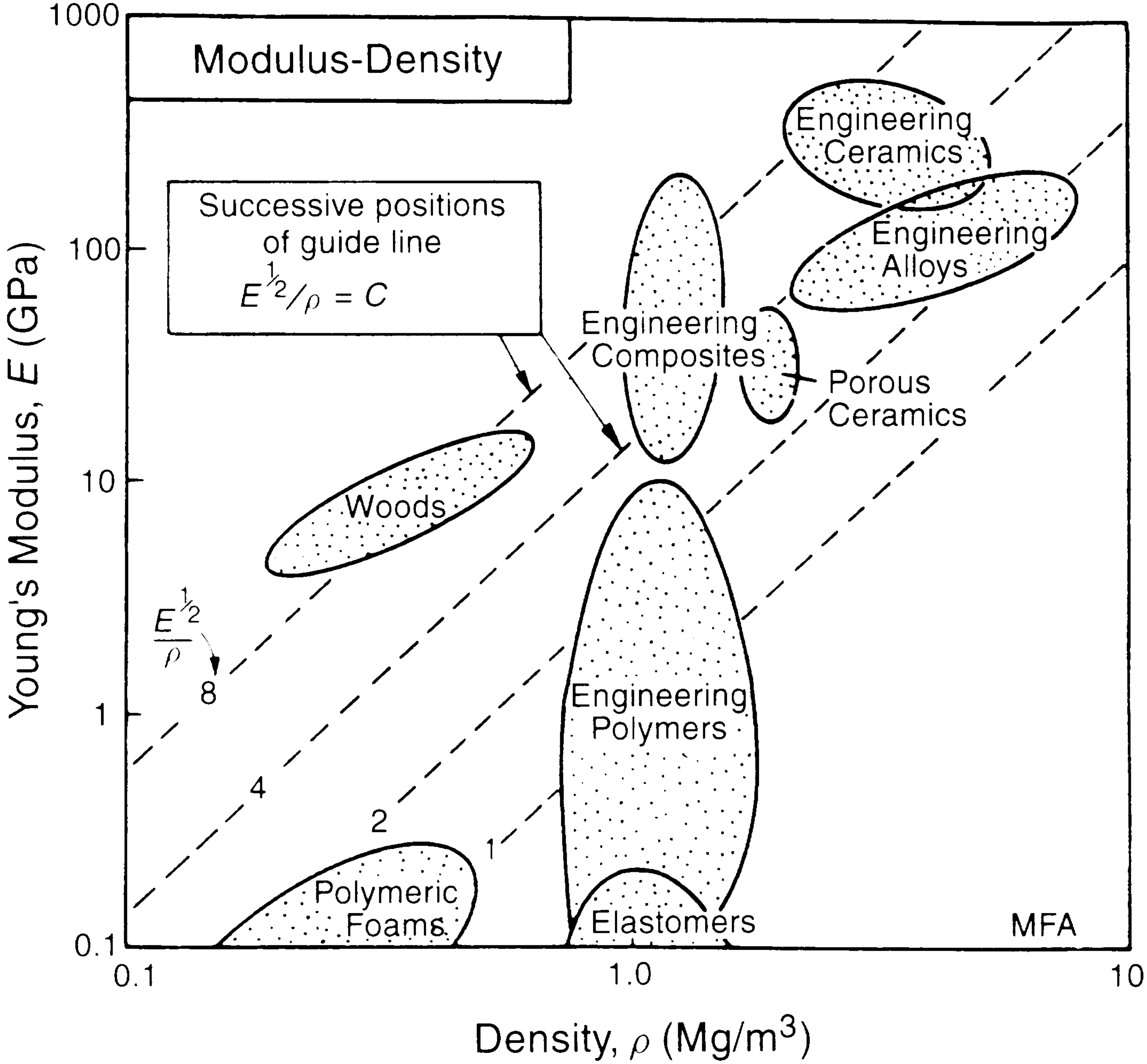
Step	*Action*
1	*Define the design requirements:* (a) Function: what does the component do? (b) Objective: what is to be maximized or minimized? (c) Constraints: essential requirements which must be met: stiffness, strength, corrosion resistance, forming characteristics...
2	Develop an *equation* for the objective in terms of the functional requirements, the geometry and the material properties (the *objective function*).
3	Identify the *free* (unspecified) *variables*.
4	Develop *equations* for the constraints (no yield; no fracture; no buckling, etc.).
5	*Substitute* for the free variables from the constraint equations into the objective function.
6	*Group the variables* into three groups: functional requirements, F, geometry, G, and material properties, M, thus Performance characteristic $\leq f_1(F) f_2(G) f_3(M)$ or Performance characteristic $\geq f_1(F) f_2(G) f_3(M)$
7	*Read off* the material index, expressed as a quantity M, which optimizes the performance characteristic.

Examples of material indices

Function, Objective and Constraint	*Index*
Tie, minimum weight, stiffness prescribed	$\frac{E}{\rho}$
Beam, minimum weight, stiffness prescribed	$\frac{E^{1/2}}{\rho}$
Beam, minimum weight, strength prescribed	$\frac{\sigma_y^{2/3}}{\rho}$
Beam, minimum cost, stiffness prescribed	$\frac{E^{1/2}}{C_m \rho}$
Beam, minimum cost, strength prescribed	$\frac{\sigma_y^{2/3}}{C_m \rho}$
Column, minimum cost, buckling load prescribed	$\frac{E^{1/2}}{C_m \rho}$
Spring, minimum weight for given energy storage	$\frac{\sigma_y^2}{E \rho}$
Thermal insulation, minimum cost, heat flux prescribed	$\frac{1}{\lambda C_m \rho}$
Electromagnet, maximum field, temperature rise prescribed	$\kappa C_p \rho$

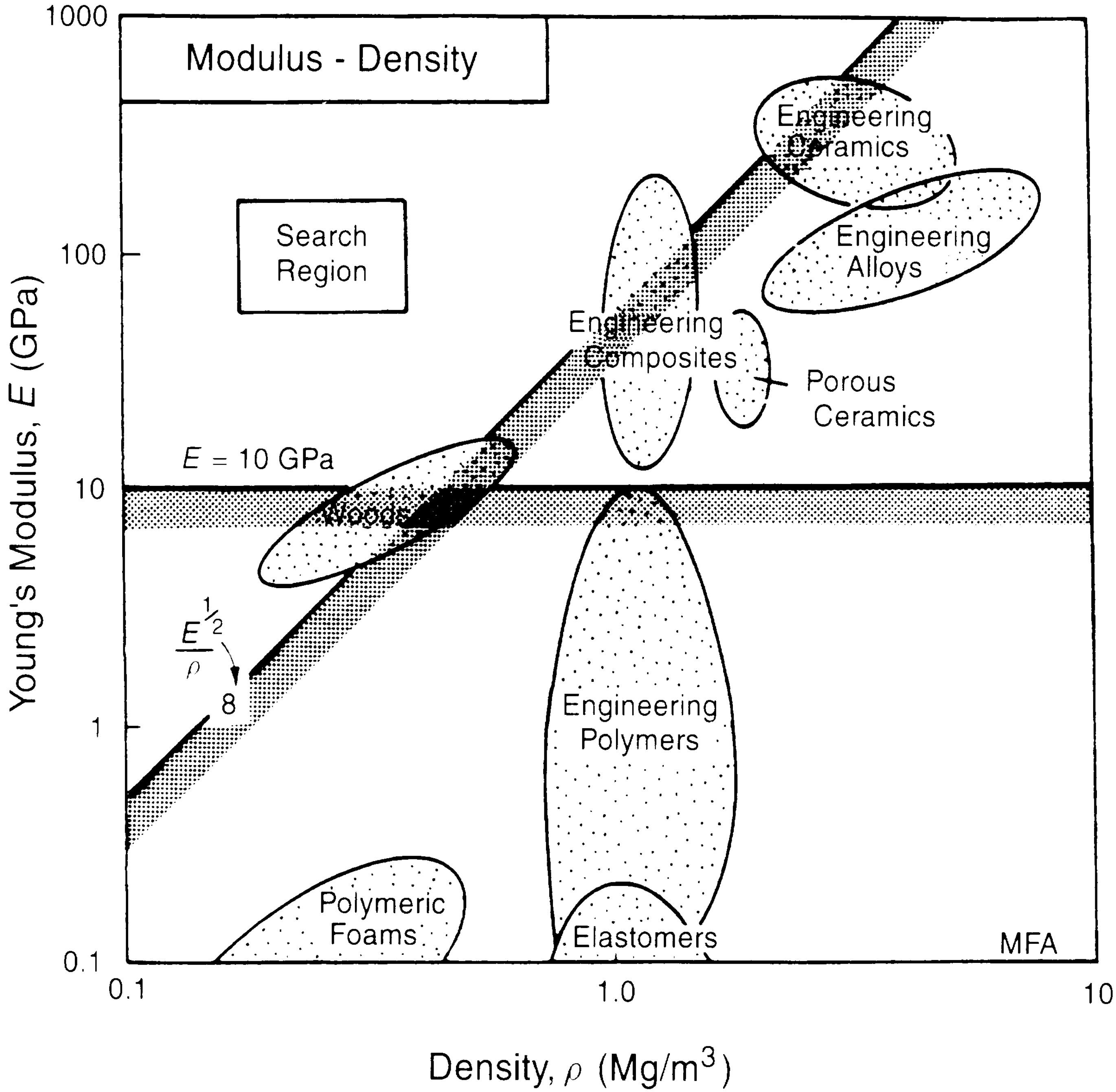
(ρ = density; E = Young's modulus; σ_y = elastic limit; C_m = cost/kg; λ = thermal conductivity; κ = electrical conductivity; C_p = specific heat)

Once the material index has been determined, it is relatively easy to read off the subset of materials which maximize performance for each loading geometry. For example, for light stiff beams, all the materials which lie on a line of constant $M = E^{1/2}/\rho$ perform equally well. Those above the line are better, and those below the line are worse. The next figure shows a grid of lines corresponding to values $M = E^{1/2}/\rho$ from 1 to 8 in appropriate units.


Modulus-Density
Successive positions
of guide line
$E^{1/2}/\rho = C$
$\frac{E^{1/2}}{\rho}$
8
4
2
1
Woods
Polymeric
Foams
Engineering
Composites
Engineering
Polymers
Elastomers
Porous
Ceramics
Engineering
Ceramics
Engineering
Alloys
MFA
1000
100
10
1
0.1
0.1
1.0
10
Young's Modulus, E (GPa)
Density, ρ (Mg/m3)

The subset of materials which particularly good values a of the index is identified by picking a line which isolates a **search area** containing a small number of candidates.

Property limits can be added, narrowing the search window: that corresponding to $E > 10$ Gpa is shown.


Modulus - Density
Search Region
Engineering Ceramics
Engineering Alloys
Engineering Composites
Porous Ceramics
E = 10 GPa
Woods
E^1/2 / ρ
8
Engineering Polymers
Polymeric Foams
Elastomers
MFA
1000
100
10
1
0.1
0.1
1.0
10
Young's Modulus, E (GPa)
Density, ρ (Mg/m3)

CASE STUDY: Structural Materials For Buildings

- The most expensive thing that most people buy is the house they live in.

- Roughly half the cost of the house is the cost of the materials from which it is built.

- The materials are used in large quantities, around 200 tons for a single family house.

The materials are used in three ways:

- Structurally to hold the building up.
- Cladding, to keep the weather out.
- Internals: plumbing, heating, kitchen etc.

We shall consider the selection of materials for the structure. They must be:

- **Stiff** so that the building does not flex too much under wind loads or internal loading.

- **Strong** so that it does not collapse.

- **Cheap** since such a lot of material is used.

Design goal: Stiffness and strength at minimum cost!

To be be more specific, we consider the selection of material for floor beams (joists), and columns.

Design requirements for floor beams:

- **Function**: Floor Beams and Columns.

- **Objective**: Minimize the cost

- **Constraints**:

  1. Length L specified.

  2. Stiffness: must not deflect too much under design loads.

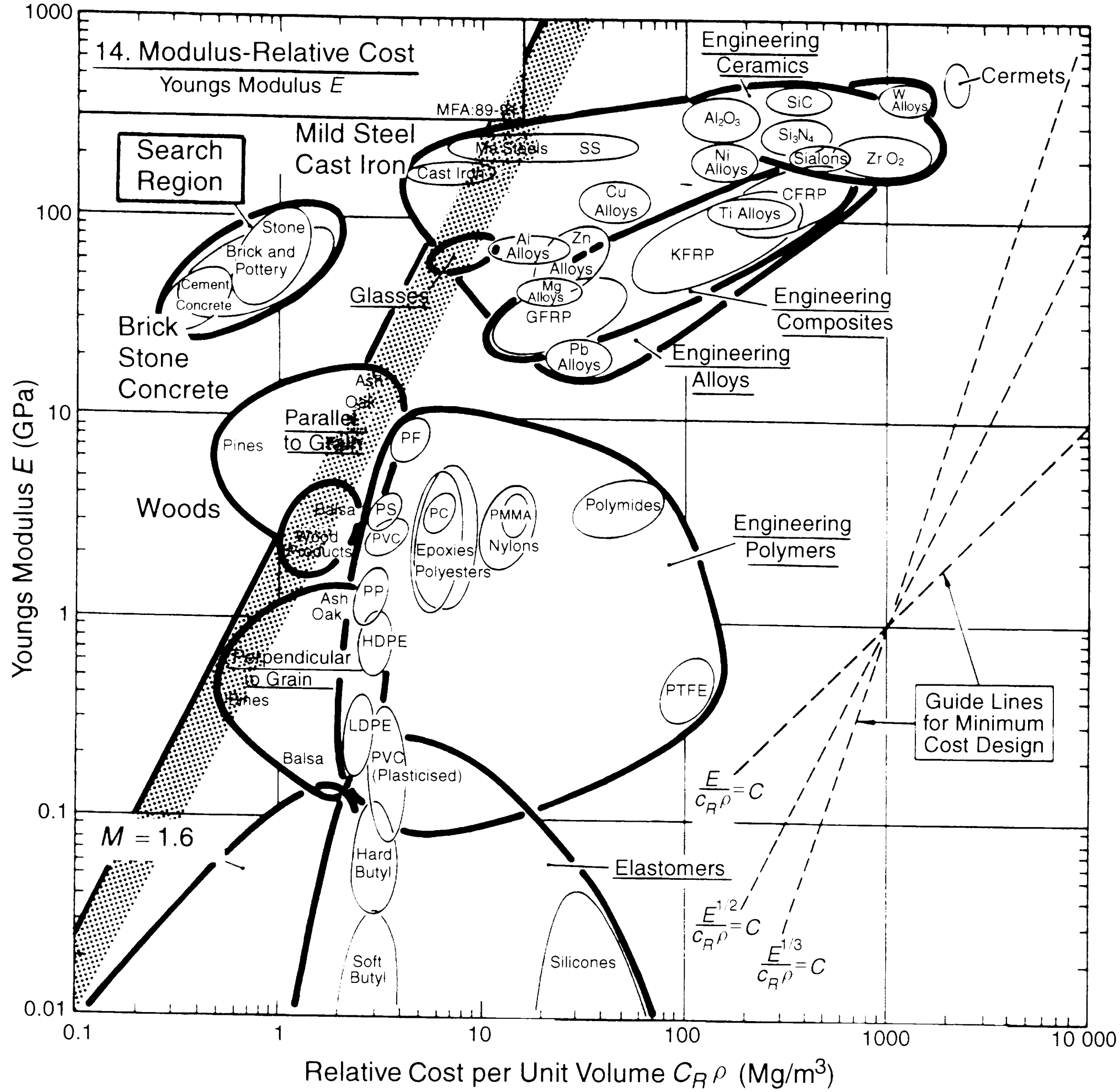
  3. Strength: must not fail under design loads.

The two indices we want to maximize are

$$\boxed{M_1 = \frac{E^{1/2}}{C_m \rho}}$$

and

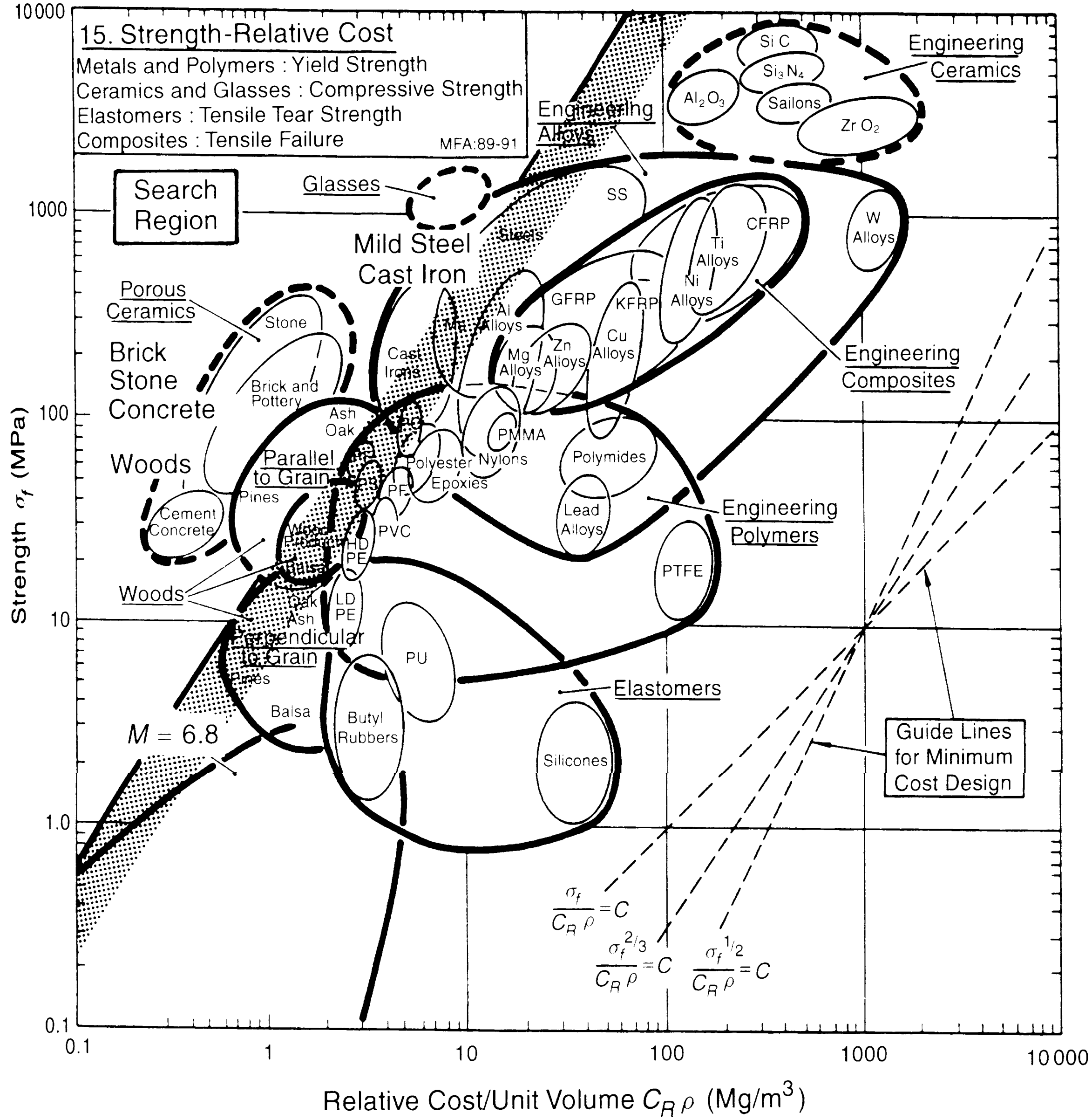
$$\boxed{M_2 = \frac{\sigma_f^{2/3}}{C_m \rho}}$$



The selection of cheap, stiff materials for the structural frames of buildings.

The shaded area in the E versus $C_R\rho$ chart isolates:

- Brick, stone, concrete
- Softwoods
- Cast iron and the cheaper steels



The selection of cheap, strong materials for the structural frames of buildings.

The shaded area in the σ_f versus $C_R\rho$ chart isolates almost the same selection:

- Brick, stone, concrete
- Softwoods
- Cast iron and the cheaper steels

These are exactly the materials from which buildings have been made in the past, and are currently being made from!

Material	M_1 GPa$^{1/2}$/ (k\$/m^3)	M_2 MPa$^{2/3}$/ (k\$/m^3)	Comment
Concrete	40	80	Use in compression only
Brick	20	45	"
Stone	15	45	"
Woods	15	80	Tension or Compression
Cast Iron	5	20	freedom of section shape
Steel	3	21	"
Reinforced Conc.	20	60	"

Civil construction (buildings, bridges, roads, etc.) is material intensive. The cost of materials dominates. Only the cheapest materials qualify, and the design must be adapted to use them!