SMA ADVANCED MATERIALS PROGRAM
MATERIALS SELECTION, DESIGN AND ECONOMICS
SMA 5103 & MIT 3.57
FALL 1999

MATERIALS SELECTION IN MECHANICAL DESIGN

PART 2

L. Anand

MATERIAL SELECTION CHARTS

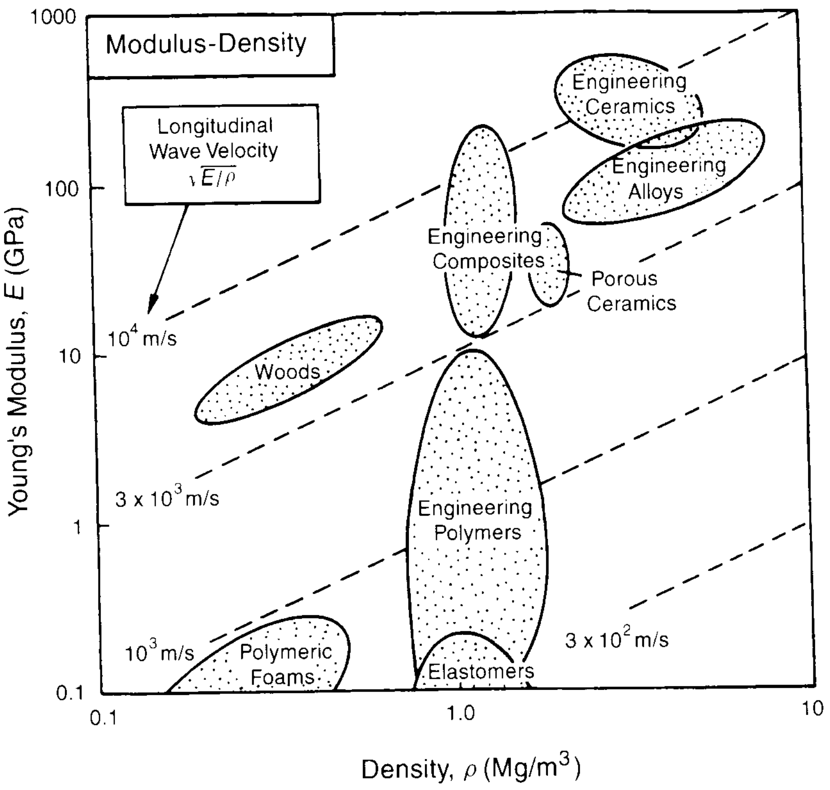
- Material properties limit performance.

- A quick method is needed for surveying the values of the design-limiting properties.

- Usually it is a combination of properties that matters.

  For example in the design for lightweight components the ratios E/ρ and σ_f/ρ are of significance.

- This suggests the idea of plotting one property against another, and mapping out the fields in various two-dimensional property spaces, e.g. E versus ρ, occupied by each material class.



$$c = \sqrt{\frac{E}{\rho}}, \text{ longitudinal wave speed in a rod}$$

$$\log E = 2 \log c + \log \rho.$$

Class	*Members*	*Short name*
Engineering Alloys (The metals and alloys of engineering)	Aluminium alloys	Al alloys
	Copper alloys	Cu alloys
	Lead alloys	Lead alloys
	Magnesium alloys	Mg alloys
	Molybdenum alloys	Mo alloys
	Nickel alloys	Ni alloys
	Steels	Steels
	Tin alloys	Tin alloys
	Titanium alloys	Ti alloys
	Tungsten alloys	W alloys
	Zinc alloys	Zn alloys
Engineering Polymers (The thermoplastics and thermosets of engineering)	Epoxies	EP
	Melamines	MEL
	Polycarbonate	PC
	Polyesters	PEST
	Polyethylene, high density	HDPE
	Polyethylene, low density	LDPE
	Polyformaldehyde	PF
	Polymethylmethacrylate	PMMA
	Polypropylene	PP
	Polytetrafluorethylene	PTFE
	Polyvinylchloride	PVC
Engineering Ceramics (Fine ceramics capable of load-bearing application)	Alumina	Al_2O_3
	Diamond	C
	Sialons	Sialons
	Silicon Carbide	SiC
	Silicon Nitride	Si_3N_4
	Zirconia	ZrO_2
Engineering Composites (The composites of engineering practice.) A distinction is drawn between the properties of a ply — 'UNIPLY' — and of a laminate — 'LAMINATES'	Carbon fibre reinforced polymer	CFRP
	Glass fibre reinforced polymer	GFRP
	Kevlar fibre reinforced polymer	KFRP
Porous Ceramics (Traditional ceramics, cements, rocks and minerals)	Brick	Brick
	Cement	Cement
	Common rocks	Rocks
	Concrete	Concrete
	Porcelain	Pcln
	Pottery	Pot
Glasses (Ordinary silicate glass)	Borosilicate glass	B-glass
	Soda glass	Na-glass
	Silica	SiO_2
Woods (Separate envelopes describe properties parallel to the grain and normal to it, and wood products)	Ash	Ash
	Balsa	Balsa
	Fir	Fir
	Oak	Oak
	Pine	Pine
	Wood products (ply, etc)	Woods

(continued overleaf)

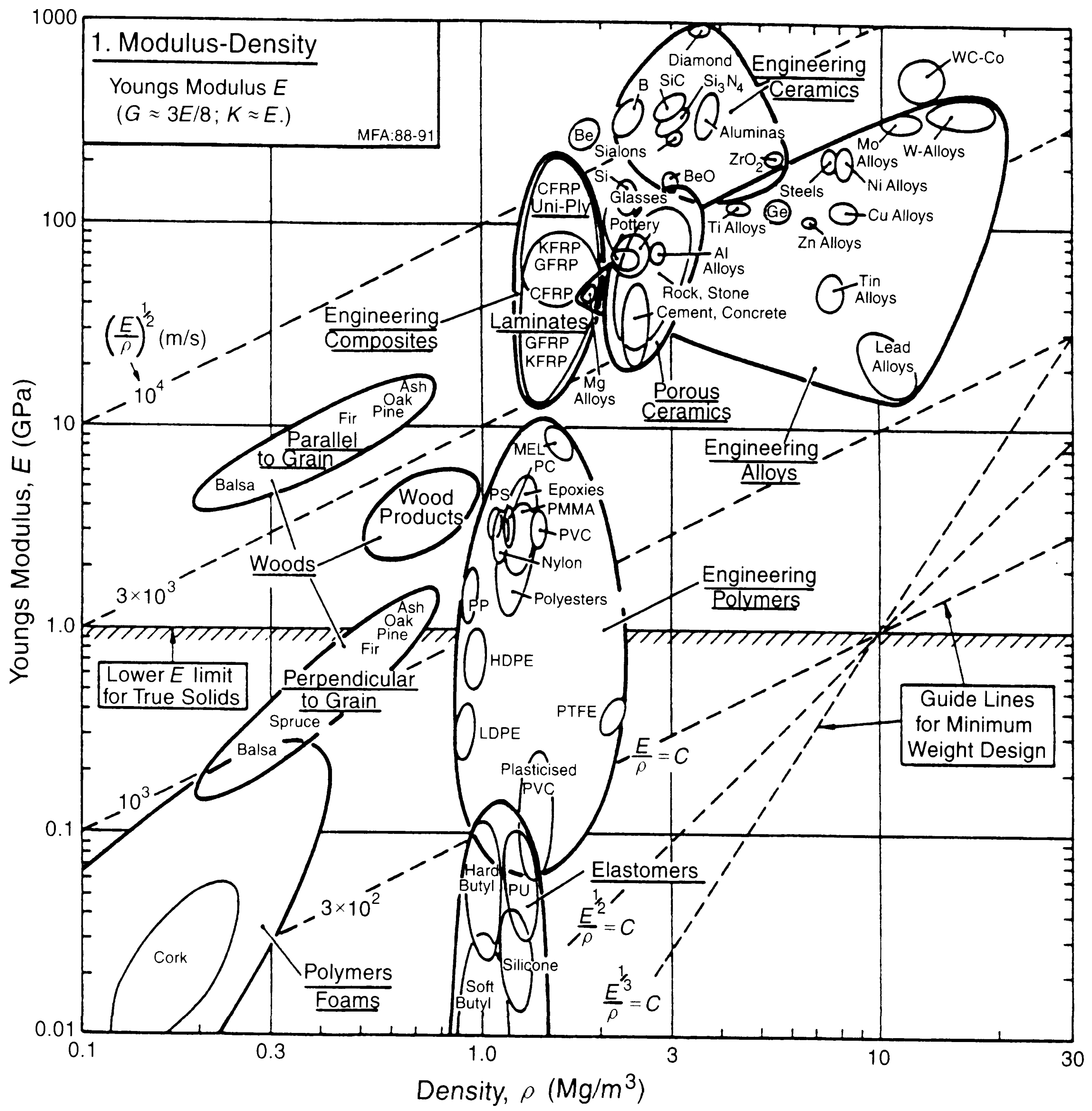
Class	*Members*	*Short name*
Elastomers	Natural rubber	Rubber
(Natural and artificial rubbers)	Hard Butyl rubber	Hard Butyl
	Polyurethanes	PU
	Silicone rubber	Silicone
	Soft Butyl rubber	Soft Butyl
Polymer Foams	These include:	
(Foamed polymers of	Cork	Cork
engineering)	Polyester	PEST
	Polystyrene	PS
	Polyurethane	PU

Data for Young's modulus, *E*

Material	*E* (GN m^{-2})	*Material*	*E* (GN m^{-2})
Diamond	1000	Niobium and alloys	80–110
Tungsten carbide, WC	450–650	Silicon	107
Osmium	551	Zirconium and alloys	96
Cobalt/tungsten carbide cermets	400–530	Silica glass, SiO_2 (quartz)	94
Borides of Ti, Zr, Hf	450–500	Zinc and alloys	43–96
Silicon carbide, SiC	430–445	Gold	82
Boron	441	Calcite (marble, limestone)	70–82
Tungsten and alloys	380–411	Aluminium	69
Alumina, Al_2O_3	385–392	Aluminium and alloys	69–79
Beryllia, BeO	375–385	Silver	76
Titanium carbide, TiC	370–380	Soda glass	69
Tantalum carbide, TaC	360–375	Alkali halides (NaCl, LiF, etc.)	15–68
Molybdenum and alloys	320–365	Granite (Westerly granite)	62
Niobium carbide, NbC	320–340	Tin and alloys	41–53
Silicon nitride, Si_3N_4	280–310	Concrete, cement	30–50
Beryllium and alloys	290–318	Fibreglass (glass-fibre/epoxy)	35–45
Chromium	285–290	Magnesium and alloys	41–45
Magnesia, MgO	240–275	GFRP	7–45
Cobalt and alloys	200–248	Calcite (marble, limestone)	31
Zirconia, ZrO_2	160–241	Graphite	27
Nickel	214	Shale (oil shale)	18
Nickel alloys	130–234	Common woods, $\parallel$ to grain	9–16
CFRP	70–200	Lead and alloys	16–18
Iron	196	Alkyds	14–17
Iron-based super-alloys	193–214	Ice, H_2O	9.1
Ferritic steels, low-alloy steels	196–207	Melamines	6–7
Stainless austenitic steels	190–200	Polyimides	3–5
Mild steel	200	Polyesters	1.8–3.5
Cast irons	170–190	Acrylics	1.6–3.4
Tantalum and alloys	150–186	Nylon	2–4
Platinum	172	PMMA	3.4
Uranium	172	Polystyrene	3–3.4
Boron/epoxy composites	80–160	Epoxies	2.6–3
Copper	124	Polycarbonate	2.6
Copper alloys	120–150	Common woods, $\perp$ to grain	0.6–1.0
Mullite	145	Polypropylene	0.9
Vanadium	130	PVC	0.2–0.8
Titanium	116	Polyethylene, high density	0.7
Titanium alloys	80–130	Foamed polyurethane	0.01–0.06
Palladium	124	Polyethylene, low density	0.2
Brasses and bronzes	103–124	Rubbers	0.01–0.1
		Foamed polymers	0.001–0.01

Data for density, ρ

Material	ρ *(Mg m^{-3})*	*Material*	ρ *(Mg m^{-3})*
Osmium	22.7	Silicon carbide, SiC	2.5–3.2
Platinum	21.4	Silicon nitride, Si_3N_4	3.2
Tungsten and alloys	13.4–19.6	Mullite	3.2
Gold	19.3	Beryllia, BeO	3.0
Uranium	18.9	Common rocks	2.2–3.0
Tungsten carbide, WC	14.0–17.0	Calcite (marble, limestone)	2.7
Tantalum and alloys	16.6–16.9	Aluminium	2.7
Molybdenum and alloys	10.0–13.7	Aluminium alloys	2.6–2.9
Cobalt/tungsten-carbide cermets	11.0–12.5	Silica glass, SiO_2 (quartz)	2.6
Lead and alloys	10.7–11.3	Soda glass	2.5
Silver	10.5	Concrete/cement	2.4–2.5
Niobium and alloys	7.9–10.5	GFRPs	1.4–2.2
Nickel	8.9	Carbon fibres	2.2
Nickel alloys	7.8–9.2	PTFE	2.3
Cobalt and alloys	8.1–9.1	Boron fibre/epoxy	2.0
Copper	8.9	Beryllium and alloys	1.85–1.9
Copper alloys	7.5–9.0	Magnesium and alloys	1.74–1.88
Brasses and bronzes	7.2–8.9	Fibreglass (GFRP/Polyester)	1.55–1.95
Iron	7.9	Graphite, high strength	1.8
Iron-based super-alloys	7.9–8.3	PVC	1.3–1.6
Stainless steels, austenitic	7.5–8.1	CFRPs	1.5–1.6
Tin and alloys	7.3–8.0	Polyesters	1.1–1.5
Low-alloy steels	7.8–7.85	Polyimides	1.4
Mild steel	7.8–7.85	Epoxies	1.1–1.4
Stainless steel, ferritic	7.5–7.7	Polyurethane	1.1–1.3
Cast iron	6.9–7.8	Polycarbonate	1.2–1.3
Titanium carbide, TiC	7.2	PMMA	1.2
Zinc and alloys	5.2–7.2	Nylon	1.1–1.2
Chromium	7.2	Polystyrene	1.0–1.1
Zirconium carbide, ZrC	6.6	Polyethylene, high-density	0.94–0.97
Zirconium and alloys	6.6	Ice, H_2O	0.92
Titanium	4.5	Natural rubber	0.83–0.91
Titanium alloys	4.3–5.1	Polyethylene, low-density	0.91
Alumina, Al_2O_3	3.9	Polypropylene	0.88–0.91
Alkali halides	3.1–3.6	Common woods	0.4–0.8
Magnesia, MgO	3.5	Foamed plastics	0.01–0.6
		Foamed polyurethane	0.06–0.2


1. Modulus-Density
Youngs Modulus E
($G \approx 3E/8$; $K \approx E$.)
MFA:88-91
Youngs Modulus, E (GPa)
Density, ρ (Mg/m^3)
1000
100
10
1.0
0.1
0.01
0.1
0.3
1.0
3
10
30
Diamond
Engineering Ceramics
WC-Co
B
SiC
Si_3N_4
Aluminas
Be
Sialons
Si
BeO
Glasses
ZrO_2
Mo Alloys
W-Alloys
Steels
Ni Alloys
Ti Alloys
Ge
Cu Alloys
Zn Alloys
Pottery
Al Alloys
Tin Alloys
Rock, Stone
Cement, Concrete
Lead Alloys
CFRP Uni-Ply
KFRP
GFRP
CFRP
Laminates
GFRP
KFRP
Engineering Composites
Mg Alloys
Porous Ceramics
Engineering Alloys
$\left(\frac{E}{\rho}\right)^{\frac{1}{2}}$ (m/s)
10^4
Ash
Oak
Pine
Fir
Parallel to Grain
Balsa
MEL
PC
Epoxies
PS
PMMA
PVC
Nylon
Polyesters
Wood Products
Woods
3×10^3
Engineering Polymers
PP
HDPE
Ash
Oak
Pine
Fir
Perpendicular to Grain
Lower E limit for True Solids
Spruce
Balsa
LDPE
PTFE
Guide Lines for Minimum Weight Design
$\frac{E}{\rho} = C$
10^3
Plasticised PVC
Hard Butyl
PU
Elastomers
3×10^2
$\frac{E^{\frac{1}{2}}}{\rho} = C$
Cork
Polymers Foams
Silicone
Soft Butyl
$\frac{E^{\frac{1}{3}}}{\rho} = C$

EXAMPLE

Select a material for a solid cylindrical rod of length L to withstand a compressive force F without buckling. It is to be of minimum mass.

Design requirements for a light column:

- **Function**: Column
- **Objective**: Minimize the mass
- **Constraint**: F specified; L specified; Must not buckle

Minimize

$$m = A\,L\,\rho \quad \text{Objective function,}$$

subject to

$$F \leq F_{cr} \quad \text{Constraint,}$$

where A is the cross-sectional area of the column and

$$F_{cr} = c\,\frac{\pi^2 E I}{L^2},$$

where

$$I = \frac{\pi R^4}{4} = \frac{A^2}{4\pi}$$

is the area moment of inertia, and c is the end-fixity coefficient (this is not important here!)

$$F \le F_{cr} \quad \Rightarrow \quad F \le c\,\frac{\pi^2 EI}{L^2} \quad \Rightarrow \quad F \le c\,\frac{\pi^2 E}{L^2}\frac{A^2}{4\pi}$$

or

$$A^2 \ge 4F\left(\frac{L^2}{c\pi}\right)\frac{1}{E} \quad \Rightarrow \quad A \ge 2F^{1/2}\left(\frac{L^2}{c\pi}\right)^{1/2}\frac{1}{E^{1/2}}$$

Since L is fixed, to minimize mass choose

$$A = 2F^{1/2}\left(\frac{L^2}{c\pi}\right)^{1/2}\frac{1}{E^{1/2}}.$$

Subsitute this in the objective function

$$m = \left\{\left(\frac{2}{\pi^{1/2}}\right)F^{1/2}\right\}\left\{\left(\frac{L^4}{c}\right)^{1/2}\right\}\left\{\left(\frac{\rho}{E^{1/2}}\right)\right\}$$

$$m = \{\text{Functional Req.}\}\,\{\text{Specified Geom.}\}\,\{\text{Material Properties}\}$$

Thus the mass will be minimized by selecting materials with the largest value of the index

$$\boxed{M = \frac{E^{1/2}}{\rho}} \qquad \text{Material Index}$$

This plots as a line

$$\log E = 2 \log M + 2 \log \rho,$$

of slope 2 on the ($\log E$) versus ($\log \rho$) chart.

Yield strength, σ_y, tensile strength, σ_{TS}, and tensile ductility, ϵ_f

Material	$\sigma_y/MN\,m^{-2}$	$\sigma_{TS}/MN\,m^{-2}$	ϵ_f
Diamond	50 000	–	0
Silicon carbide, SiC	10 000	–	0
Silicon nitride, Si_3N_4	8 000	–	0
Silica glass, SiO_2	7 200	–	0
Tungsten carbide, WC	6 000	–	0
Niobium carbide, NbC	6 000	–	0
Alumina, Al_2O_3	5 000	–	0
Beryllia, BeO	4 000	–	0
Mullite	4 000	–	0
Titanium carbide, TiC	4 000	–	0
Zirconium carbide, ZrC	4 000	–	0
Tantalum carbide, TaC	4 000	–	0
Zirconia, ZrO_2	4 000	–	0
Soda glass (standard)	3 600	–	0
Magnesia, MgO	3 000	–	0
Cobalt and alloys	180–2000	500–2500	0.01–6
Low-alloy steels (water-quenched and tempered)	500–1980	680–2400	0.02–0.3
Pressure-vessel steels	1500–1900	1500–2000	0.3–0.6
Stainless steels, austenitic	286–500	760–1280	0.45–0.65
Boron/epoxy composites (tension–compression)	–	725–1730	–
Nickel alloys	200–1600	400–2000	0.01–0.6
Nickel	70	400	0.65
Tungsten	1 000	1510	0.01–0.6
Molybdenum and alloys	560–1450	665–1650	0.01–0.36
Titanium and alloys	180–1320	300–1400	0.06–0.3
Carbon steels (water-quenched and tempered)	260–1300	500–1880	0.2–0.3
Tantalum and alloys	330–1090	400–1100	0.01–0.4
Cast irons	220–1030	400–1200	0–0.18
Copper alloys	60–960	250–1000	0.01–0.55
Copper	60	400	0.55
Cobalt/tungsten carbide cermets	400–900	900	0.02
CFRPs (tension–compression)	–	670–640	–
Brasses and bronzes	70–640	230–890	0.01–0.7
Aluminium alloys	100–627	300–700	0.05–0.3
Aluminium	40	200	0.5
Stainless steels, ferritic	240–400	500–800	0.15–0.25
Zinc alloys	160–421	200–500	0.1–1.0
Concrete, steel reinforced (tension or compression)	–	410	0.02
Alkali halides	200–350	–	0
Zirconium and alloys	100–365	240–440	0.24–0.37
Mild steel	220	430	0.18–0.25
Iron	50	200	0.3
Magnesium alloys	80–300	125–380	0.06–0.20
GFRPs	–	100–300	–
Beryllium and alloys	34–276	380–620	0.02–0.10
Gold	40	220	0.5
PMMA	60–110	110	0.03–0.05
Epoxies	30–100	30–120	–
Polyimides	52–90	–	–

Material	$\sigma_y/\mathrm{MN\,m^{-2}}$	$\sigma_{TS}/\mathrm{MN\,m^{-2}}$	ϵ_f
Nylons	49–87	100	–
Ice	85	–	0
Pure ductile metals	20–80	200–400	0.5–1.5
Polystyrene	34–70	40–70	–
Silver	55	300	0.6
ABS/polycarbonate	55	60	–
Common woods (compression, $\parallel$ to grain)	–	35–55	–
Lead and alloys	11–55	14–70	0.2–0.8
Acrylic/PVC	45–48	–	–
Tin and alloys	7–45	14–60	0.3–0.7
Polypropylene	19–36	33–36	–
Polyurethane	26–31	58	–
Polyethylene, high density	20–30	37	–
Concrete, non-reinforced, compression	20–30	–	0
Natural rubber	–	30	5.0
Polyethylene, low density	6–20	20	–
Common woods (compression, $\perp$ to grain)	–	4–10	–
Ultrapure f.c.c. metals	1–10	200–400	1–2
Foamed polymers, rigid	0.2–10	0.2–10	0.1–1
Polyurethane foam	1	1	0.1–1

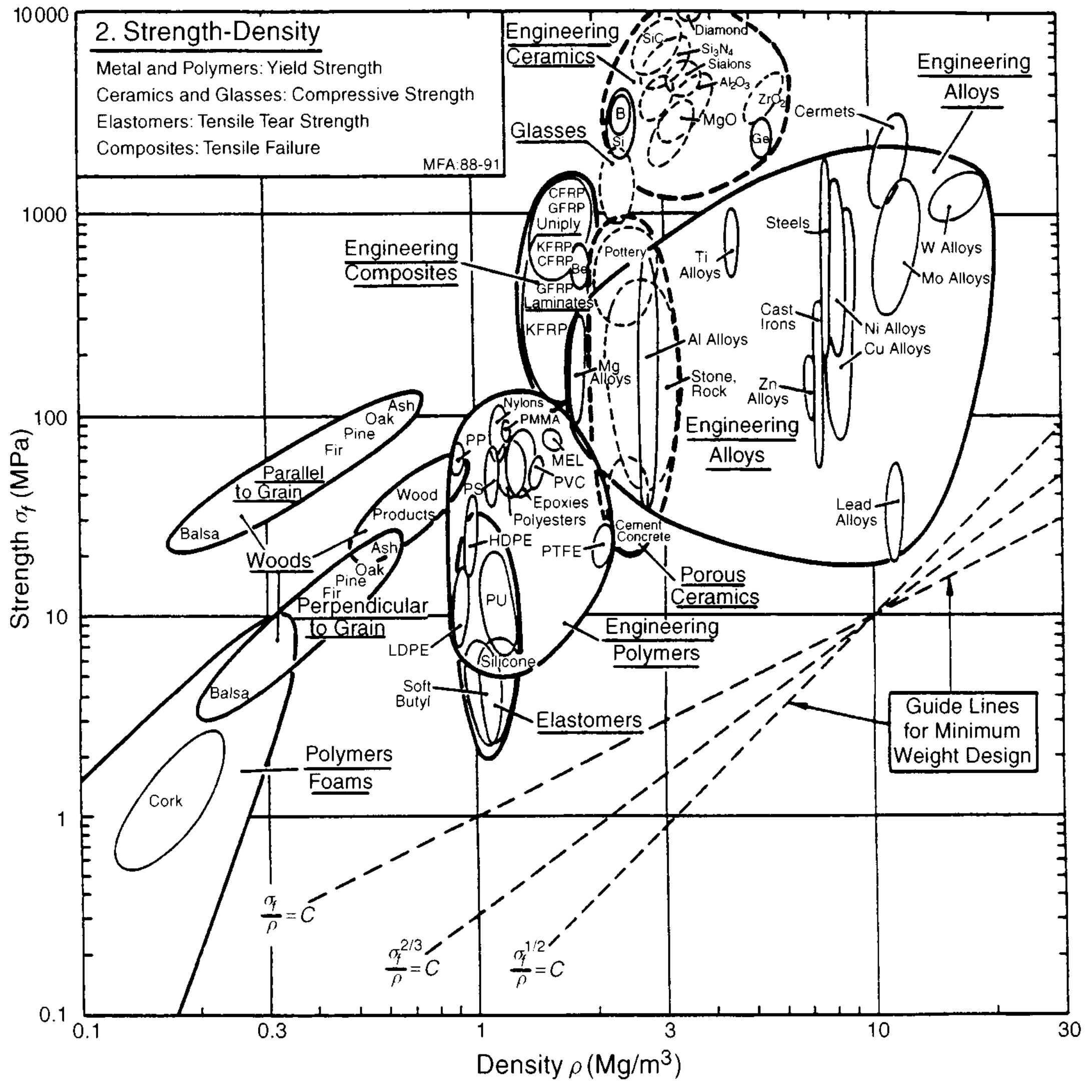
- The modulus of a material is a well-defined quantity with a sharp value. The strength is not.

- Recall that for metals and polymers σ_f is the yield strength. Since the range of materials includes those that have been worked, the range spans from initial yield to ultimate strength.

  For brittle ceramics the compressive crushing strength is plotted. Recall that this is typical 15 times the tensile strength.

  For composites it is the tensile strength which is plotted. Recall that this could be up to 30% higher than the compressive strength.

- The considerable vertical extension of a strength bubble for a given material reflects the wide variability which may be caused by alloying, working, grain size, porosity etc.


2. Strength-Density
Metal and Polymers: Yield Strength
Ceramics and Glasses: Compressive Strength
Elastomers: Tensile Tear Strength
Composites: Tensile Failure
MFA:88-91
Engineering Ceramics
SiC
Diamond
Si3N4
Sialons
Al2O3
ZrO2
MgO
Ge
B
Si
Glasses
Cermets
Engineering Alloys
CFRP
GFRP
Uniply
KFRP
CFRP
Be
GFRP
Laminates
KFRP
Pottery
Engineering Composites
Ti Alloys
Steels
W Alloys
Mo Alloys
Cast Irons
Ni Alloys
Cu Alloys
Al Alloys
Mg Alloys
Stone, Rock
Zn Alloys
Engineering Alloys
Lead Alloys
Nylons
PMMA
PP
MEL
PVC
Epoxies
Polyesters
PS
HDPE
PTFE
Cement Concrete
Porous Ceramics
PU
LDPE
Silicone
Engineering Polymers
Soft Butyl
Elastomers
Ash
Oak
Pine
Fir
Parallel to Grain
Balsa
Wood Products
Woods
Ash
Oak
Pine
Fir
Perpendicular to Grain
Balsa
Polymers Foams
Cork
Guide Lines for Minimum Weight Design
σf/ρ = C
σf^2/3/ρ = C
σf^1/2/ρ = C
Strength σf (MPa)
10000
1000
100
10
1
0.1
Density ρ (Mg/m3)
0.1
0.3
1
3
10
30

EXAMPLE

Select a material for a thin-walled spherical pressure vessel of a given radius R which must contain a pressure p, must be as light as possible, and must not fail by plastic yielding. Structures can also fail by fracture, by fatigue, and by corrosion superimposed on these other modes of failure. However, for now assume that failure by plastic yielding is the only problem.

Design requirements for a light pressure vessel:

- **Function**: Spherical pressure vessel.
- **Objective**: Minimize the mass
- **Constraints**: R specified; p specified; Must not fail by yielding

Minimize

$$m = 4\pi R^2 t\rho \quad \text{Objective function,}$$

subject to

$$\bar{\sigma} \leq \sigma_f \quad \text{Constraint,}$$

where t is the wall-thickness area of the pressure vessel and

$$\bar{\sigma} \equiv \sqrt{\frac{1}{2}\left\{(\sigma_{rr} - \sigma_{\theta\theta})^2 + (\sigma_{\theta\theta} - \sigma_{\phi\phi})^2 + (\sigma_{\phi\phi} - \sigma_{rr})^2\right\}}$$

is the equivalent tensile stress (for no shear stress case). For a thin-walled pressure vessel

$$\sigma_{\theta\theta} = \sigma_{\phi\phi} = \frac{pR}{2t}, \qquad \sigma_{rr} \approx 0.$$

Hence, in this case

$$\bar{\sigma} = \frac{pR}{2t}$$

and the constraint becomes

$$\frac{pR}{2t} \leq \sigma_f \quad \Rightarrow \quad t \geq \frac{pR}{2\sigma_f}$$

To minimize mass choose

$$t = \frac{pR}{2\sigma_f}$$

Subsitute this in the objective function

$m = \{(2\pi)\, p\} \left\{R^3\right\} \left\{\left(\frac{\rho}{\sigma_f}\right)\right\}$

$m = \{\text{Functional Req.}\} \{\text{Specified Geom.}\} \{\text{Material Properties}\}$

Thus the mass will be minimized by selecting materials with the largest value of the index

$$\boxed{M = \frac{\sigma_f}{\rho}} \qquad \text{Material Index}$$

This plots as a line

$$\log \sigma_f = \log M + \log \rho,$$

of slope 1 on the $(\log \sigma_f)$ versus $(\log \rho)$ chart.

Consider the smaller list of candidate materials given below:

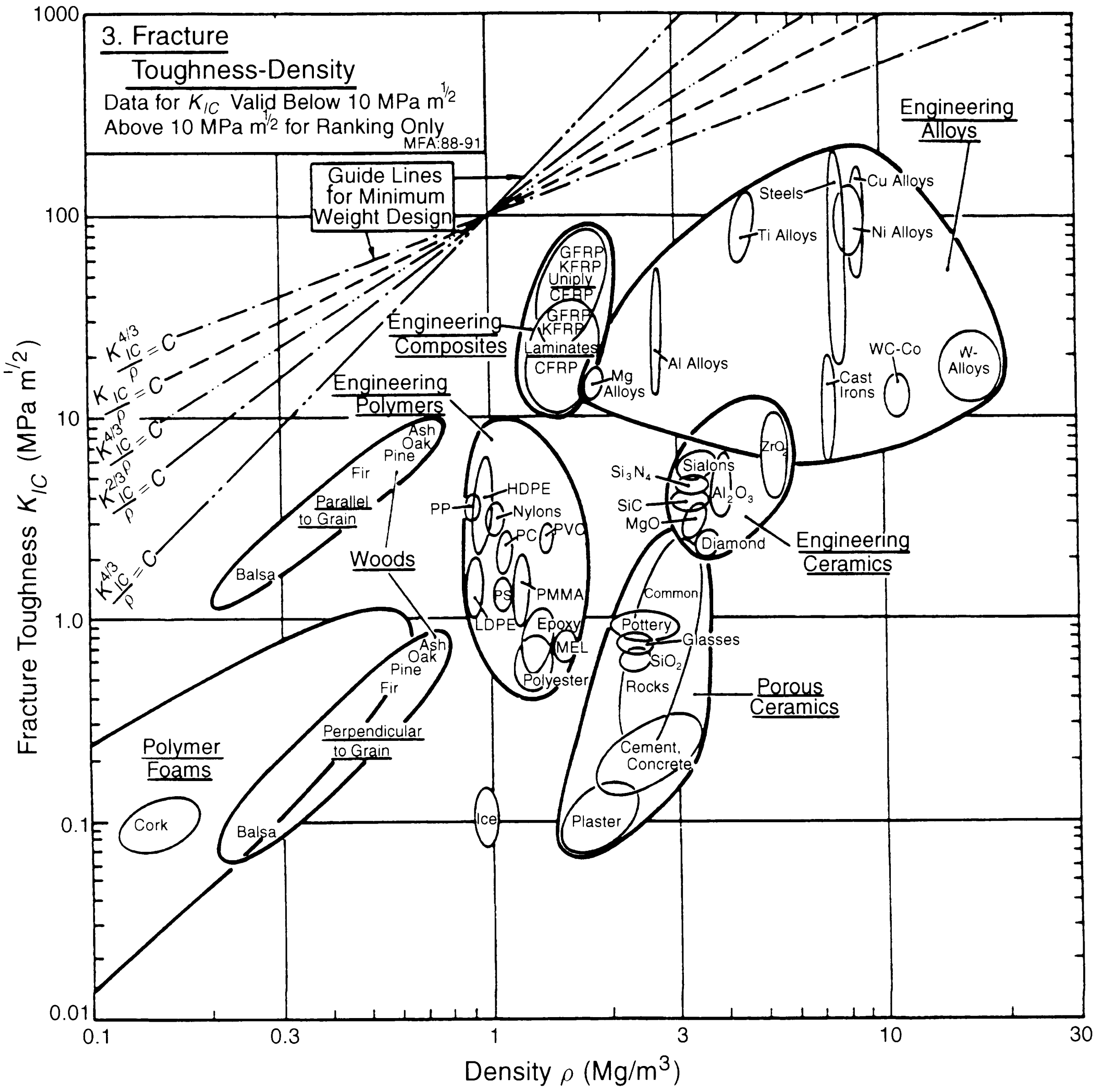
Material	σ_f MPa	ρ Mg/m^3	$\frac{\sigma_f}{\rho}$ Nm/g
Alloy steel (pressure vessel steel)	1000	7.8	128.2
Mild steel	220	7.8	28.2
Aluminum alloy	400	2.7	148.1
Titanium alloy	1000	4.5	222.2
CFRP	600	1.5	400

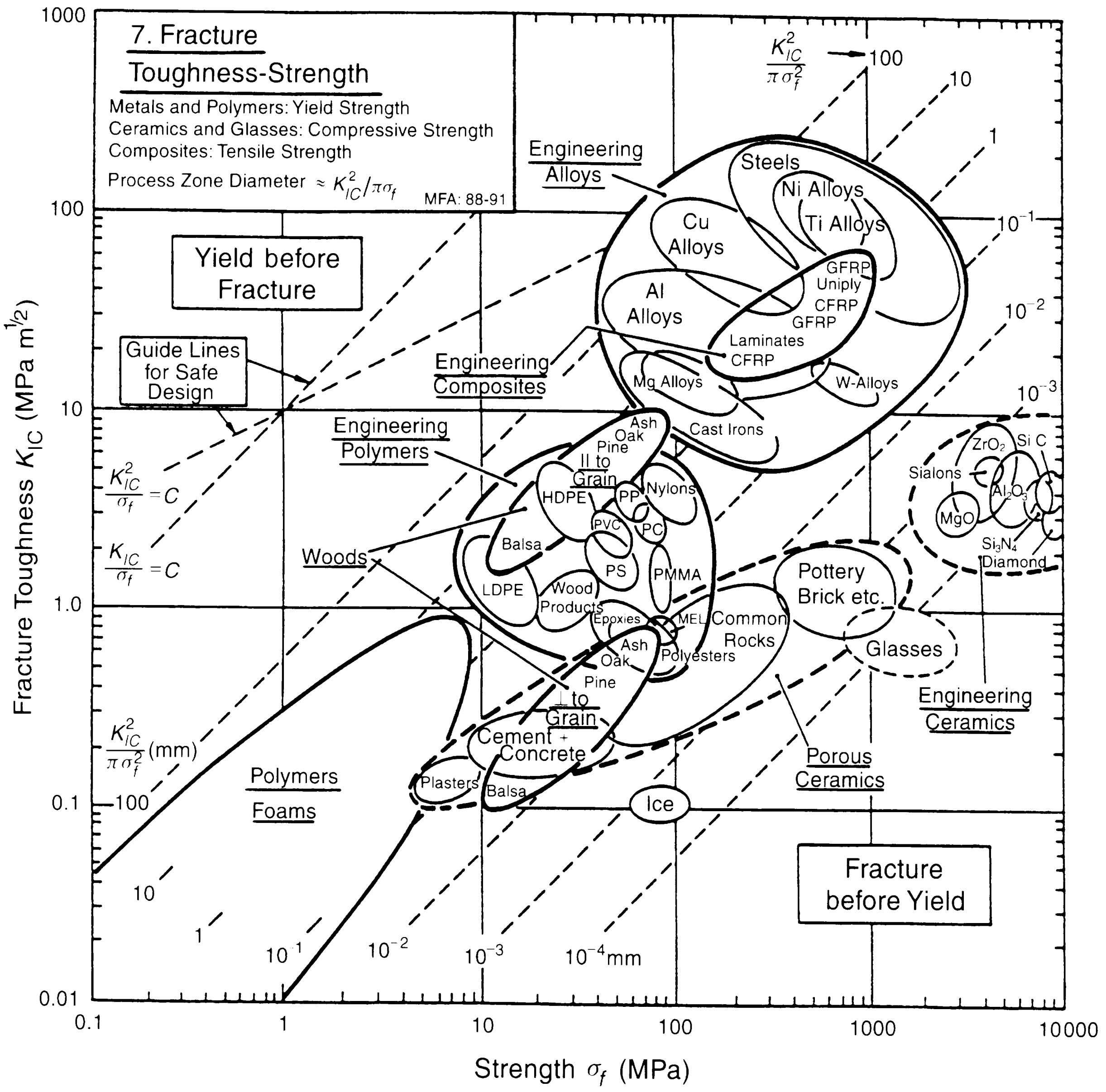
By far the lightest pressure vessel is that made of CFRP. Titanium comes next. Aluminum alloy and pressure vessel steel are further down the line.

Toughness, G_c, and fracture toughness, K_c

Material	G_c *(kJ/m^2)*	K_c *(MN/m$^{3/2}$)*
Pure ductile metals (e.g. Cu, Ni, Ag, Al)	100–1000	100–350
Rotor steels (A533; Discalloy)	220–240	204–214
Pressure-vessel steels (HY130)	150	170
High-strength steels (HSS)	15–118	50–154
Mild steel	100	140
Titanium alloys (Ti6Al4V)	26–114	55–115
GFRPs	10–100	20–60
Fibreglass (glassfibre epoxy)	40–100	42–60
Aluminium alloys (high strength–low strength)	8–30	23–45
CFRPs	5–30	32–45
Common woods, crack ⊥ to grain	8–20	11–13
Boron-fibre epoxy	17	46
Medium-carbon steel	13	51
Polypropylene	8	3
Polyethylene (low density)	6–7	1
Polyethylene (high density)	6–7	2
ABS polystyrene	5	4
Nylon	2–4	3
Steel-reinforced cement	0.2–4	10–15
Cast iron	0.2–3	6–20
Polystyrene	2	2
Common woods, crack ∥ to grain	0.5–2	0.5–1
Polycarbonate	0.4–1	1.0–2.6
Cobalt/tungsten carbide cermets	0.3–0.5	14–16
PMMA	0.3–0.4	0.9–1.4
Epoxy	0.1–0.3	0.3–0.5
Granite (Westerly Granite)	0.1	3
Polyester	0.1	0.5
Silicon nitride, Si_3N_4	0.1	4–5
Beryllium	0.08	4
Silicon carbide SiC	0.05	3
Magnesia, MgO	0.04	3
Cement/concrete, unreinforced	0.03	0.2
Calcite (marble, limestone)	0.02	0.9
Alumina, Al_2O_3	0.02	3–5
Shale (oilshale)	0.02	0.6
Soda glass	0.01	0.7–0.8
Electrical porcelain	0.01	1
Ice	0.003	0.2*

*Values at room temperature unless starred.


3. Fracture
Toughness-Density
Data for K_{IC} Valid Below 10 MPa m$^{1/2}$
Above 10 MPa m$^{1/2}$ for Ranking Only
MFA:88-91
Guide Lines for Minimum Weight Design
$\frac{K_{IC}^{4/3}}{\rho} = C$
$\frac{K_{IC}}{\rho} = C$
$\frac{K_{IC}^{4/3}}{\rho} = C$
$\frac{K_{IC}^{2/3}}{\rho} = C$
$\frac{K_{IC}^{4/3}}{\rho} = C$
Engineering Alloys
Steels
Cu Alloys
Ti Alloys
Ni Alloys
Al Alloys
WC-Co
W-Alloys
Cast Irons
Mg Alloys
GFRP
KFRP
Uniply
CFRP
GFRP
KFRP
Laminates
CFRP
Engineering Composites
Engineering Polymers
Ash
Oak
Pine
Fir
Parallel to Grain
Balsa
Woods
HDPE
PP
Nylons
PC
PVC
PS
PMMA
LDPE
Epoxy
MEL
Polyester
Sialons
Si$_3$N$_4$
ZrO$_2$
Al$_2$O$_3$
SiC
MgO
Diamond
Engineering Ceramics
Common
Pottery
Glasses
SiO$_2$
Rocks
Porous Ceramics
Cement, Concrete
Plaster
Ice
Ash
Oak
Pine
Fir
Perpendicular to Grain
Balsa
Polymer Foams
Cork
1000
100
10
1.0
0.1
0.01
Fracture Toughness K_{IC} (MPa m$^{1/2}$)
0.1
0.3
1
3
10
30
Density ρ (Mg/m^3)


7. Fracture Toughness-Strength
Metals and Polymers: Yield Strength
Ceramics and Glasses: Compressive Strength
Composites: Tensile Strength
Process Zone Diameter ≈ $K_{IC}^2/\pi\sigma_f$
MFA: 88-91
Yield before Fracture
Fracture before Yield
Guide Lines for Safe Design
$\frac{K_{IC}^2}{\sigma_f} = C$
$\frac{K_{IC}}{\sigma_f} = C$
$\frac{K_{IC}^2}{\pi\sigma_f^2}$ (mm)
$\frac{K_{IC}^2}{\pi\sigma_f^2}$ → 100
100
10
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}mm
Engineering Alloys
Steels
Ni Alloys
Ti Alloys
Cu Alloys
Al Alloys
GFRP Uniply CFRP
GFRP
Laminates CFRP
Engineering Composites
Mg Alloys
W-Alloys
Cast Irons
Engineering Polymers
Ash Oak Pine
II to Grain
HDPE
PP
Nylons
PVC
PC
Balsa
PS
LDPE
Wood Products
PMMA
Woods
Epoxies
MEL
Polyesters
Ash Oak Pine
⊥ to Grain
Common Rocks
Pottery Brick etc.
Glasses
Cement + Concrete
Plasters
Balsa
Ice
Polymers Foams
Porous Ceramics
Engineering Ceramics
Sialons
ZrO₂
Si C
MgO
Al₂O₃
Si₃N₄
Diamond
Fracture Toughness K_{IC} (MPa $m^{1/2}$)
1000
100
10
1.0
0.1
0.01
Strength σ_f (MPa)
0.1
1
10
100
1000
10000

EXAMPLE

Pressure vessels, from the simplest aerosol can to the biggest boiler, are designed for safety based on either of the following two criteria: (a) Yield before break, and (b) Leak before break. **Small pressure vessels** are usually designed to allow general yield at a pressure too low to propagate any crack the vessel may contain — "yield before break"; the distortion caused by yielding is easy to detect and the pressure can be safely released. With **large pressure vessels** this may not be possible; instead, safe design is achieved by ensuring that the smallest crack that will propagate to cause brittle fracture of the vessel has a depth greater than the wall thickness of the pressure vessel — "leak before break"; the leak is easily detected and it releases the pressure gradually and thus safely.

Design requirements for a safe pressure vessel:

- **Function**: Spherical pressure vessel
- **Objective**: Maximum safety with maximum p.
- **Constraints**: R specified. Also
  - Must yield before break or
  - Must leak before break
  - Wall thickness small to reduce mass and cost

As a simple model of a pressure vessel consider a **thin-walled** cylindrical pressure vessel with hemispherical caps. The vessel has a mean radius of R, a wall thickness of t and is subjected to an internal pressure p

1. For a thin-walled pressure vessel

   $$\sigma_{\theta\theta} = \sigma_{\phi\phi} = \frac{pR}{2t}, \qquad \sigma_{rr} \approx 0.$$

   Hence, in this case

   $$\bar{\sigma} = \frac{pR}{2t} \quad \text{equivalent tensile stress}$$

   and

   $$\sigma_1 = \frac{pR}{2t} \quad \text{maximum principal stress}$$

2. Assume that if a crack is present, then it is of a semi-elliptical shape of length $2a$ and depth a through the wall thickness. Consider the design to prevent brittle fracture of a material with toughness K_{Ic}:

$$K_I \leq K_{Ic} \quad \Rightarrow \quad Q\,\sigma_1\sqrt{\pi a} \leq K_{Ic} \quad \Rightarrow \quad Q\,\frac{pR}{2t}\sqrt{\pi a} \leq K_{Ic}$$

or

$$\frac{pR}{2t} \leq \frac{K_{Ic}}{Q\sqrt{\pi}}\frac{1}{\sqrt{a}}$$

3. Assume that no crack is present and consider the design to prevent yielding of a material with strength σ_f:

$$\bar{\sigma} \leq \sigma_f \quad \Rightarrow \quad \frac{pR}{2t} \leq \sigma_f.$$

4. The value of the crack length a^* of the crack depth where the curve delineating the area safe against fracture interects the line delineating the curve safe aginst failure due to yielding is

$$a^* = \frac{1}{Q^2\pi}\left(\frac{K_{Ic}}{\sigma_f}\right)^2$$

The physical significance of a^* is that if $a < a^*$, then the crack will not extend even if the the stress $\left(\frac{pR}{2t}\right)$ reaches the boundary for failure due to yielding. The pressure vessel will deform stably in manner which can be detected — "yield before break". The quantity a^* can be thought of a tolerable crack size for the "yield before break" design philosophy.

The material

$$\boxed{M_1 = \frac{K_{Ic}}{\sigma_f}}$$

which when maximized, maximizes the tolerable crack size a^*.

5. Consider large pressure vessel of given radius R whose wall thickness t has been designed to contain a pressure p without yielding. That is,

$$t > \frac{pR}{2\sigma_f}.$$

   Large pressure vessels cannot always be non-destructively tested by X-rays or ultrasonically, and proof testing them may be impractical. Further, cracks can grow because of cyclic loading or corrosion, so that a single examination at the beginning of service life is not sufficient. Under these situations safety can be ensured by arranging that a crack large enough to penetrate through the wall thickness is still stable (that is it does not satisfy the criterion for brittle fracture), and that the leak caused by the crack penetrating the wall can be detected — "leak before break".

For a pressure vessel of given radius R whose wall thickness has been designed such that

$$t > \frac{pR}{2\sigma_f},$$

this can be achieved if we additionally require that

$$t < a^* = \frac{1}{Q^2\pi}\left(\frac{K_{Ic}}{\sigma_f}\right)^2$$

For a pressure vessel of a given radius R which has been designed not to fail by yielding, and which should leak before it breaks, the maximum pressure is carried when

$$\frac{pR}{2\sigma_f} = \frac{1}{Q^2\pi}\left(\frac{K_{Ic}}{\sigma_f}\right)^2 \quad \Rightarrow \quad Q^2\pi\frac{pR}{2} = \left(\frac{K_{Ic}^2}{\sigma_f}\right).$$

Thus the maximum pressure is carried most safely by the material with the greatest value of

$$\boxed{M_2 = \left(\frac{K_{Ic}^2}{\sigma_f}\right)}.$$

We require that the wall of the vessel must also be as thin as possible for reasons of economy of material and to keep the vessel light. From

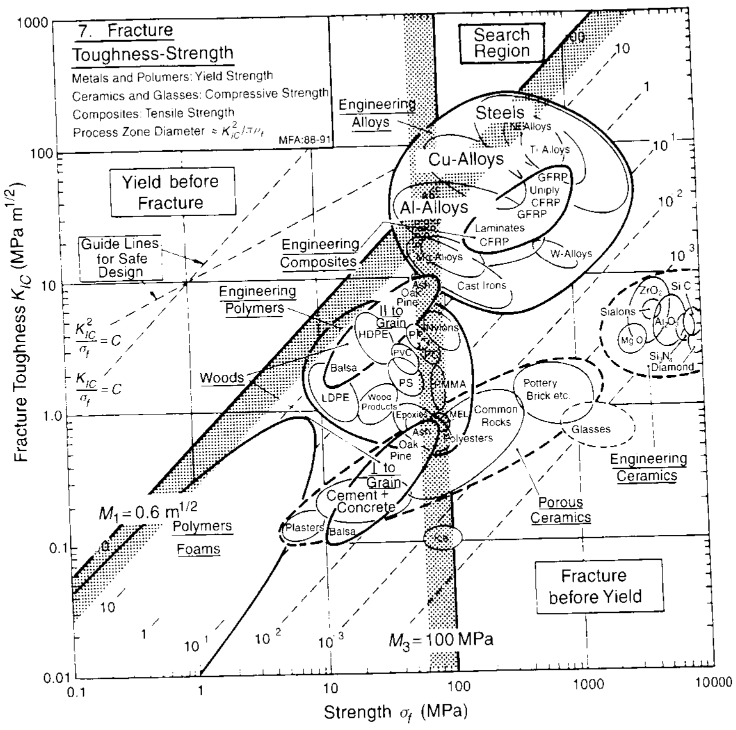
$$t > \frac{pR}{2\sigma_f},$$

the thinnest wall is that with the largest value of σ_f.

Thus we also wish to maximize

$$\boxed{M_3 = \sigma_f}.$$

6. The material performance indices $M_1 = \dfrac{K_{Ic}}{\sigma_f}$, $M_2 = \dfrac{K_{Ic}^2}{\sigma_f}$ and $M_3 = \sigma_f$, appear as lines of slope 1, 1/2, and as lines that are vertical on the $\log K_{Ic}$ versus $\log \sigma_f$ plot. The figure delineates the search region for $M_1 \geq 0.6$; this excludes everything but the toughest steels, copper and aluminum alloys. A second selection line at $M_3 = 100$ eliminates aluminum alloys.



- Large pressure vessels are always made of steel
- Those for models are made from copper.
- The alternative criterion $M_2 = \frac{K_{Ic}^2}{\sigma_f}$ favors steel more strongly, but it does not change the conclusions.

Materials for safe pressure vessels

Material	$M_1 = \frac{K_{Ic}}{\sigma_f}$ $(m^{1/2})$	$M_3 = \sigma_f$ *(MPa)*	*Comment*
Tough steels	>0.6	300	These are the pressure-vessel steels, standard in this application.
Tough copper alloys	>0.6	120	OFHC Hard drawn copper.
Tough Al-alloys	>0.6	80	1000 and 3000 series Al-alloys.
Ti-alloys	02	700	High yield but low safety margin.
High-strength Al-alloys	0.1	500	Good for light pressure vessels.
GFRP/CFRP	0.1	500	