Multiattribute Utility

- Objective: to present a practical method to obtain U(X)
- Motivation
- Axiomatic Basis
- Procedure
- Formula

Motivation

- Curse of Dimensionality
  - Procedure for 1-dimensional utility function can, in theory, be applied to an *n*-dimensional utility function
  - But, consider the number of points to be assessed if we divide a range of N dimensions into quarters

Dimensions	Number of points
1	5 - 2 = 3
2	(5)(5) - 2 = 23
3	(5)(5)(5) - 2 = 123

Axiomatic Basis

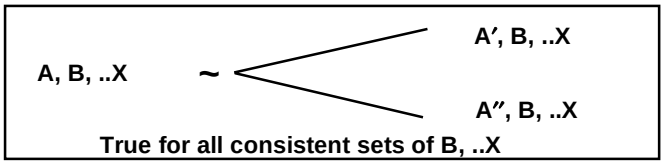
- **Preferential Independence - an ordinal condition**
  - **The order of preference between any 2 pairs of outcomes is constant, regardless of level of other outcomes**
  - **If $(X_1', X_2') > (X_1'', X_2'')$ for any $(X_3', ... X_N')$**
  - **Example**

    I prefer (1 cup coffee, black) to (2 cups coffee, w/ sugar), regardless of wealth
  - **Consequence**

    Can compare dimensions two at a time, independently of others

Automatic Basis (cont'd)

- **Utility Independence - a cardinal condition**
  - **The relative intensity of value for different amounts of one type of outcome is independent of level of all other outcomes.**



Axiomatic Basis (cont'd)

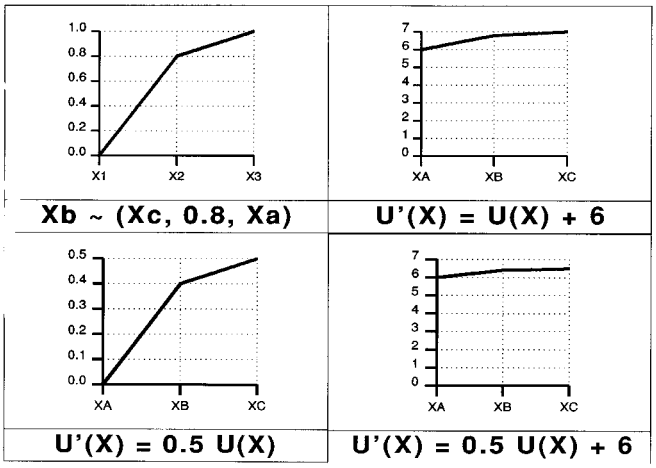
- Example
  - » When hungry, I prefer 1 plate of food for sure to a 50:50 gamble on 2 plates or none, regardless of noise.
- Consequence
  - » Can assess $U(X_i)$ once, and use it for all cross sections of U(X), subject to a positive linear transformation
  - » "Shape" of $U(X_i)$ constant

Note Concerning "Constancy of Shape"

- To say that a utility function retains its "shape" means that the utility function can only undergo a constant linear transformation, i.e. $U'(\underline{X}) = a\,U(\underline{X}) + b$

Note Concerning "Constancy of Shape" (cont'd)

Examples



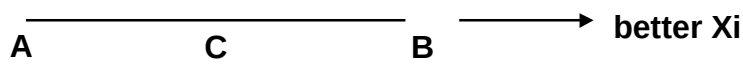
Procedure for U(X)

- **Establish the range of each dimension - X_i^* to X_i**
- **Set** $X_* = (X_{1*}, ..., X_{n*})$ $U(X_*) = 0$
  $X^* = (\text{all the best})$ $U(X^*) = 1$
- **Establish the relative value of each dimension:**

$$(X_{*1}, ...X_i^*,...X_{*n}) \sim \begin{cases} k_i & X_{\text{all best}} \\ & X_{\text{all worst}} \end{cases}$$

Procedure for U(X) (cont'd)

- Estimate 1-dimensional $U(X_i)$; scale from 0 -> 1 for each case
- Scale 1-dimensional $U(X_i)$ into U(X) Between any two points in X



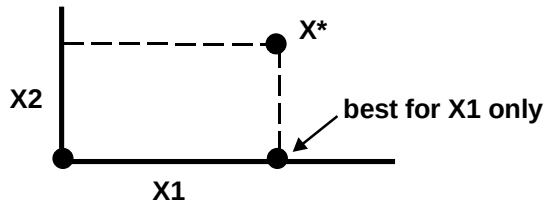
$UC = UA + p(UB - UC)$
p = proportion from $U(X_i)$

To Establish Relative Value of Each Dimension

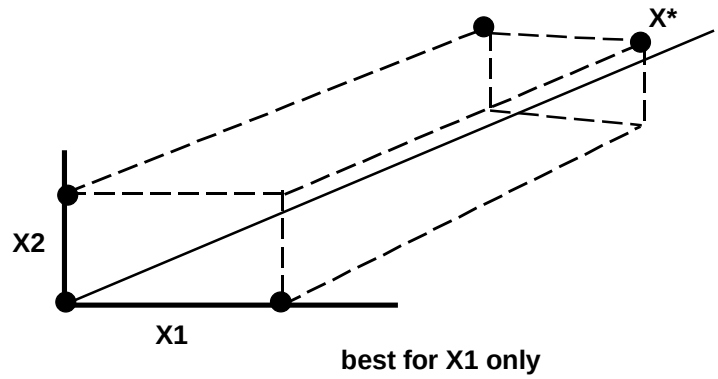
- Balancing Act between "best" and "worst" over all <u>X</u>
- For best in one dimension only

Graphically

Two Dimensions



To Establish Relative Value of Each Dimension

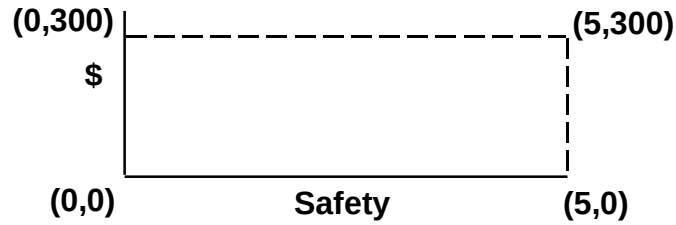


MAUA Example

U(X) X1 = Safety X2 = Profit

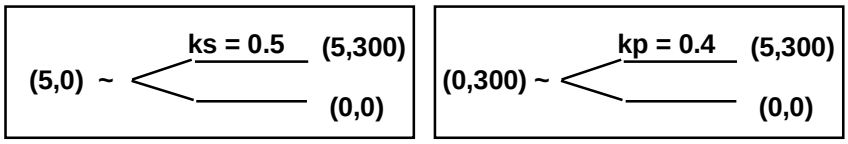
1. $X_{1*} = 0$; $X_1^* = 5$
   $X_{2*} = 0$; $X_2^* = 300$

2. U(0,0) = 0; U(5,300) = 1



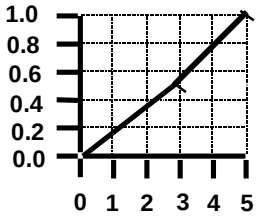
MAUA Example (cont'd)

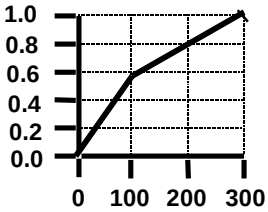
3.

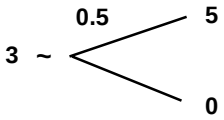


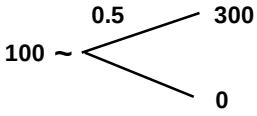
Example (cont'd)

4. Single attribute functions



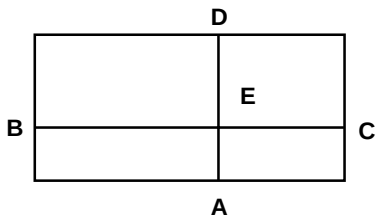






Example (cont'd)

5. **Assessments**



UA = 0 + 1/2 (0.5 - 0) = 0.25
UB = 0 + 1/2 (0.4 - 0) = 0.20
UD = 0.4 + 1/2 (1 - 0.4) = 0.70
UC = 0.5 + 1/2 (1 - 0.5) = 0.75
UE = 0.25 + 1/2 (0.45) = 0.475
= 0.2 + 1/2 (0.55) = 0.475

Formula

$$K\,U(\underline{X}) + 1 = \prod_i (K\,K_i\,U_i(X) + 1)$$

U(X) and U(X_i) all scaled between 0 and 1

- For 2 dimensions, quadratic solutions make it possible to solve directly for K
  - $K = (1 - K_1 - K_2)/K_1K_2$
- For larger numbers of dimensions, iterative solutions (e.g., Newton's method) appropriate

Formula (cont'd)

$$K\,U(\underline{X}) + 1 = \prod_i (K\,K_i\,U_i(X) + 1)$$

- **Guidelines:**

If the sum of all $k_i < 1$	$K > 0$
If the sum of all $k_i > 1$	$-1 < K < 0$
If the sum of all $k_i = 1$	$K = 0$; $U(X) = \sum K_i\,U_i(X_i)$